

# Admissibility Restoration: The Structural Necessity of Symmetry-Breaking Fields in Projected Ontology

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## Abstract

In Volume IV of this series, we showed that non-admissible gauge kernels produce structural confinement: the Lie bracket defect  $\Delta(A, B) = [A, B]$  makes color charge a fiber-non-invariant predicate, rendering it unobservable. This result applies to any non-abelian gauge theory — including  $SU(2)$ , the gauge group of the weak interaction. Yet weak charges (flavor) **are** observed, and the  $W$  and  $Z$  bosons appear as massive particles with finite range. This paper resolves the apparent contradiction within the Projected Ontology (POT) framework, without assuming quantum mechanics, a Lagrangian, or the Higgs mechanism. We introduce the concept of **admissibility restoration**: a non-admissible kernel can become effectively admissible through coupling to an additional field  $\varphi$  that compensates the Lie bracket defect. We derive three structural constraints on any restoring field: (1) it must be gauge-charged (a gauge-invariant field cannot cancel a gauge-covariant defect); (2) it must have a preferred non-zero value (a trivial field cannot restore anything); and (3) it must introduce a characteristic scale (the preferred value breaks the scale-free structure of the unrestored kernel). These three properties — gauge-charged, non-zero preferred value, characteristic scale — are exactly the defining features of the Standard Model Higgs field. We do not derive the Higgs potential, the scalar spin, the specific representation, or the 125 GeV mass; these are dynamical properties beyond the algebraic framework. What we derive is that **any** field restoring admissibility must have these three properties. The Higgs is a specific Skolem witness for this structural requirement; Technicolor and Composite Higgs models are alternative witnesses satisfying the same constraints. The result yields a three-class classification of gauge theories from a single structural principle: (1) admissible — observable charges, massless gauge boson (electromagnetism); (2) non-admissible with restoration — observable charges, massive gauge boson (weak interaction); (3) non-admissible without restoration — confined charges (strong interaction). Three forces, one classification principle, no dynamics. All results are verified for internal logical consistency by the Z3 SMT solver via 16 axiomatic examples.

**Keywords:** admissibility restoration, Higgs mechanism, spontaneous symmetry breaking, projected ontology, gauge theory classification, weak interaction, confinement, formal verification, Z3

## 1 Introduction

Volume IV of this series derived a structural necessary condition for confinement: when the projection kernel is non-admissible (the Lie bracket defect  $\Delta(A, B) = [A, B] \neq 0$ ), the kernel's

image is non-invariant on gauge orbits. By the Main Theorem of projection fiber theory, any predicate that is non-invariant on fibers cannot be determined from the projection’s image. Color charge is such a predicate. This is structural confinement.

The result applies to any non-abelian gauge theory.  $SU(3)$  is non-abelian, and the strong interaction confines — consistent with the theorem. But  $SU(2)$  is also non-abelian, and the weak interaction does **not** confine. The  $W^\pm$  and  $Z^0$  bosons are observed as massive particles with a range of approximately  $10^{-18}$  m. Weak charges (left-handed isospin) are measured in experiments. This appears to contradict the confinement theorem.

This paper resolves the contradiction. The resolution does not require abandoning the confinement theorem; it requires extending the admissibility framework. The key concept is **admissibility restoration**: a non-admissible kernel can become effectively admissible through coupling to an additional field that compensates the Lie bracket defect. We derive the structural constraints that any such restoring field must satisfy, and show that these constraints match the defining properties of the Standard Model Higgs field — without assuming its existence.

The derivation uses no quantum mechanics. No Lagrangian is written. No path integral is evaluated. No potential is assumed. The constraints emerge from the algebraic structure of the kernel and the requirement that charges be observable.

The paper is organized as follows. Section 2 recapitulates the admissibility framework and the confinement theorem. Section 3 defines the composite kernel and admissibility restoration. Section 4 derives the structural constraints on the restoring field. Section 5 establishes the mass consequence. Section 6 presents the three-class classification. Section 7 discusses the relation to the Standard Model, the Fradkin–Shenker complementarity, and prior work. Section 8 addresses falsifiability. Section 9 concludes.

## 2 Admissibility and Confinement (Recapitulation)

We summarize the results of Volumes I–IV that this paper extends.

**Admissible kernels.** A projection kernel  $K$  is admissible if it satisfies: (1) linearity —  $K(A + B) = K(A) + K(B)$ ; (2) scalar compatibility —  $K(cA) = cK(A)$ ; (3) zero preservation —  $K(0) = 0$ . These conditions formalize Occam’s Razor for projections: an admissible kernel projects without adding structure.

**The admissibility defect.** For any kernel  $K$ , the defect  $\Delta(K, A, B) = K(A + B) - K(A) - K(B)$  measures the failure of linearity. An admissible kernel has  $\Delta = 0$  for all inputs. The Yang–Mills kernel  $K_{\text{YM}}(A) = dA + A \wedge A$  has defect  $\Delta(A, B) = [A, B]$ , the Lie bracket of the gauge algebra.

**The abelian classification (Theorem 1 of Volume IV).** A gauge kernel is admissible if and only if the gauge group is abelian.  $U(1)$  (electromagnetism) is admissible;  $SU(N)$  for  $N \geq 2$  is not.

**Structural confinement (Theorem 3 of Volume IV).** If the kernel is non-admissible, its image is non-invariant on gauge orbits. Charge is a fiber-non-invariant predicate and cannot be determined from gauge-invariant observables. This is structural confinement.

**The puzzle.**  $SU(2)$  is non-abelian  $\Rightarrow$  the weak kernel is non-admissible  $\Rightarrow$  the confinement theorem applies. Yet weak charges **are** observed. Either the theorem is wrong, or something restores admissibility for the weak interaction that does not restore it for the strong interaction. This paper formalizes the second option.

### 3 Composite Kernels and Admissibility Restoration

We now define the central concept: admissibility restoration.

**Definition (Composite kernel).** Given a gauge kernel  $K$  and a field  $\varphi$ , the composite kernel  $K_{\text{eff}} = K_{\text{eff}}(K, \varphi)$  describes the effective projection when  $K$  operates in the presence of  $\varphi$ . The composite defect is defined as:

$$\Delta_{\text{eff}}(A, B) = K_{\text{eff}}(A + B) - K_{\text{eff}}(A) - K_{\text{eff}}(B)$$

$K_{\text{eff}}$  is admissible if and only if  $\Delta_{\text{eff}} = 0$  for all inputs.

**Definition (Admissibility restoration).** A field  $\varphi$  **restores admissibility** of a kernel  $K$  if: (1)  $K$  is not admissible ( $\Delta \neq 0$ ); (2) the composite kernel  $K_{\text{eff}}(K, \varphi)$  is admissible ( $\Delta_{\text{eff}} = 0$ ); and (3)  $\varphi$  is non-trivial ( $\varphi \neq 0$ ).

Condition (3) excludes vacuous restoration: the trivial field  $\varphi = 0$  cannot restore admissibility, because coupling to zero changes nothing.

**Theorem 1 (Restoration requires non-admissibility).** If  $\varphi$  restores admissibility of  $K$ , then  $K$  is not admissible. Conversely, an already-admissible kernel has no need of restoration.

**Theorem 2 (Restoration produces admissibility).** If  $\varphi$  restores admissibility of  $K$ , then the composite kernel  $K_{\text{eff}}(K, \varphi)$  is admissible: its defect vanishes for all inputs.

Both theorems are verified by Z3 in the theory file `pot_admissibility_restoration.kleis`.

**Remark (Existential vs. constructive).** The composite kernel  $K_{\text{eff}}(K, \varphi)$  is defined abstractly: we specify its defect condition ( $\Delta_{\text{eff}} = 0$ ) but do not construct its explicit form. This is deliberate. An explicit construction would require specifying the representation of  $\varphi$ , its coupling to  $A$ , and the algebraic mechanism of defect cancellation — all of which are dynamical choices that go beyond the structural framework. The standard construction is well-known: if  $\varphi$  is a scalar in the fundamental representation, the covariant derivative  $D_\mu \varphi = (\partial_\mu + gA_\mu)\varphi$  produces mass terms  $g^2 |\varphi_0|^2 A^2$  that effectively linearize the kernel at the scale  $|\varphi_0|$ . But this is one specific construction among many. Our result is that **some** construction must exist, not that it must take any particular form. The existential level is the natural stopping point for a structural argument; the constructive level requires dynamics.

### 4 Structural Constraints on the Restoring Field

We now derive the properties that any restoring field must possess. These are not assumptions — they follow from the definitions.

**Theorem 3 (The restoring field must be gauge-charged).** If  $\varphi$  restores admissibility of a non-abelian kernel  $K$ , then  $\varphi$  is not gauge-invariant: it transforms non-trivially under gauge transformations.

**Proof sketch.** The defect  $\Delta(A, B) = [A, B]$  transforms covariantly under gauge transformations:  $[A, B] \rightarrow g[A, B]g^{-1}$ . To cancel this covariant defect in the composite kernel,  $\varphi$  must itself transform under the gauge group. A gauge-invariant (singlet) field commutes with the gauge transformation and cannot compensate the adjoint-transforming defect.

**Theorem 4 (The restoring field must have a preferred non-zero value).** If  $\varphi$  restores admissibility, there exists a specific field value  $\varphi_0 \neq 0$  around which the composite kernel is admissible.

**Proof sketch.** The trivial field  $\varphi = 0$  cannot restore admissibility (by condition (3) of the definition). Therefore the restoration occurs at a specific non-zero value  $\varphi_0$ . This value is preferred’ in the structural sense: it is the value at which the composite defect vanishes.

**Corollary.** The preferred value  $\varphi_0$  breaks the gauge symmetry: since  $\varphi$  is gauge-charged (Theorem 3) and  $\varphi_0 \neq 0$ , the value  $\varphi_0$  is not invariant under the full gauge group. The gauge orbit of  $\varphi_0$  defines the manifold of equivalent preferred values.

## 5 The Mass Consequence

The preferred value  $\varphi_0$  has a further structural consequence: it introduces a characteristic scale.

**Theorem 5 (Restoration introduces a scale).** If a restoring field exists for a non-admissible kernel  $K$ , then  $K$  acquires a characteristic scale. Admissible kernels have no restoration scale; unrestored non-admissible kernels have no restoration scale.

**Structural argument.** An unrestored non-admissible kernel is scale-free: the Lie bracket  $[A, B]$  has no preferred magnitude. The defect is algebraic and operates identically at all scales. When a restoring field  $\varphi$  with preferred value  $\varphi_0 \neq 0$  is introduced, the value  $|\varphi_0|$  defines a reference scale. The composite kernel’s behavior is admissible’ at scales above  $|\varphi_0|$  and non-admissible’ at scales below it. This scale manifests as a finite range for the gauge interaction — structurally equivalent to what the dynamical framework calls the **mass** of the gauge boson.

### Comparison across classes:

- **Admissible (abelian):** No defect, no restoration needed, no scale. The gauge boson is massless. Example: the photon.
- **Non-admissible, restored:** Defect compensated by  $\varphi_0$ , scale  $\sim |\varphi_0|$ . The gauge boson has finite range. Example: the  $W^\pm$  and  $Z^0$  bosons.
- **Non-admissible, unrestored:** Defect uncompensated, no scale. Structural confinement instead of mass. Example: gluons (confined).

The scale is a structural consequence of the preferred value, not an input. We do not assume a mass term or a potential. The existence of the scale follows from the fact that  $\varphi_0 \neq 0$  breaks the scale-free structure of the bare non-admissible kernel.

## 6 The Three-Class Classification

We now state the main result: a complete classification of gauge theories from a single structural principle.

**Theorem 6 (Three-class classification of gauge theories).** Every gauge theory falls into exactly one of three classes, determined by two structural properties of its kernel:

$$\text{Class} = \begin{cases} 1 & \text{if } K \text{ is admissible (abelian)} \\ 2 & \text{if } K \text{ is non-admissible and a restoring field exists} \\ 3 & \text{if } K \text{ is non-admissible and no restoring field exists} \end{cases}$$

The observable properties of each class are:

Class	Charges	Gauge Boson	Physical Example
1: Admissible	Observable	Massless	$U(1)$ : photon
2: Restored	Observable	Massive (has scale)	$SU(2)$ : $W^\pm, Z^0$
3: Unrestored	Confined	Confined	$SU(3)$ : gluons

This classification reproduces the observed phenomenology of all three gauge interactions of the Standard Model. It is derived from two algebraic properties — admissibility and the existence of a restoring field — without assuming quantum mechanics, the Standard Model, or the Higgs mechanism.

**The role of empirical input.** The classification framework is structural: it defines three classes and derives their observable properties. But the **assignment** of specific gauge interactions to specific classes requires empirical input. The framework does not prove that  $SU(3)$  has no restoring field; it takes as empirical input that quarks are confined (no free color charges have been observed), and infers that no restoration has occurred. Similarly, it takes as empirical input that weak charges are observed ( $W$  and  $Z$  bosons are detected), and infers that a restoring field must exist. The structural contribution is the **conditional**: **if** charges are observed in a non-abelian theory, **then** a restoring field with specific properties must exist. **If** charges are confined, **then** no restoring field is present. Which conditional applies to which interaction is an empirical question. The framework provides the logic; nature provides the classification.

**The unification.** In the standard framework, electromagnetism, the weak interaction, and the strong interaction are described by separate Lagrangians with separate coupling constants, unified only at high energies through grand unification hypotheses. In the POT framework, they are three instances of a single structural classification. The difference between them is not in the dynamics but in the kernel algebra: abelian vs. non-abelian, restored vs. unrestored. Three forces, one principle.

## 7 Constructive Example: The Linearization Model

The preceding sections established the existence of a restoring field under structural constraints. We now construct an explicit, minimal model that exhibits all features of admissibility restoration, making the abstract algebra concrete and computable.

**The model.** Consider a one-parameter family of kernels on  $\mathbb{R}$ :

$$K_\alpha(x) = x + \alpha \cdot x^2, \quad \alpha \geq 0$$

For  $\alpha = 0$ , this is the identity kernel  $K_0(x) = x$ , which is linear and therefore admissible (Class 1). For  $\alpha > 0$ , the quadratic term introduces non-linearity — the algebraic analog of the  $A \wedge A$  self-coupling in Yang–Mills theory.

**The defect.** The admissibility defect is:

$$\begin{aligned} \Delta(a, b) &= K_\alpha(a + b) - K_\alpha(a) - K_\alpha(b) \\ &= 2\alpha ab \end{aligned}$$

This is non-zero whenever  $\alpha > 0$  and  $a, b \neq 0$ . The defect is proportional to the **product** of inputs — the one-dimensional analog of the Lie bracket  $[A, B]$ .

**The restoring field.** Introduce a restoring field with preferred value  $\varphi_0 > 0$ . Define the composite kernel as the linearization of  $K_\alpha$  at  $\varphi_0$ :

$$K_{\text{eff}}(x) = K'_\alpha(\varphi_0) \cdot x = (1 + 2\alpha\varphi_0) \cdot x$$

This is linear by construction. Its defect:

$$\Delta_{\text{eff}}(a, b) = K_{\text{eff}}(a + b) - K_{\text{eff}}(a) - K_{\text{eff}}(b) = 0$$

The composite kernel is admissible. Restoration is achieved.

**The mass scale.** The restored kernel has slope  $(1 + 2\alpha\varphi_0)$  rather than 1. The departure from the identity kernel defines a characteristic scale:

$$m = K'_\alpha(\varphi_0) - K'_\alpha(0) = 2\alpha\varphi_0$$

This scale is zero if  $\varphi_0 = 0$  (trivial field, no restoration) and positive if  $\varphi_0 > 0$  (non-trivial restoration). The scale  $m$  is the structural analog of the gauge boson mass.

**Verification of structural predictions.** This constructive model satisfies all five predictions from Section 8 of the theory:

1. **Gauge-charged:**  $\varphi_0$  is not invariant under  $K_\alpha$  — indeed,  $K_\alpha(\varphi_0) = \varphi_0 + \alpha\varphi_0^2 \neq \varphi_0$  for  $\alpha > 0$ .
2. **Preferred non-zero value:** restoration occurs at  $\varphi_0 > 0$ , not at  $\varphi_0 = 0$ .
3. **Scale introduced:**  $m = 2\alpha\varphi_0 > 0$ .
4. **Unrestored sector confines:** for  $\alpha > 0$  without a restoring field, the defect  $2\alpha ab$  prevents linearity — the kernel is non-admissible.
5. **Admissible sector is scale-free:** for  $\alpha = 0$ ,  $K_0(x) = x$  has no scale.

**Exact vs. local.** The composite kernel  $K_{\text{eff}}$  is not an approximation of  $K_\alpha$  — it is a **different kernel** whose defect is exactly zero, not approximately zero. The linearization constructs a new,

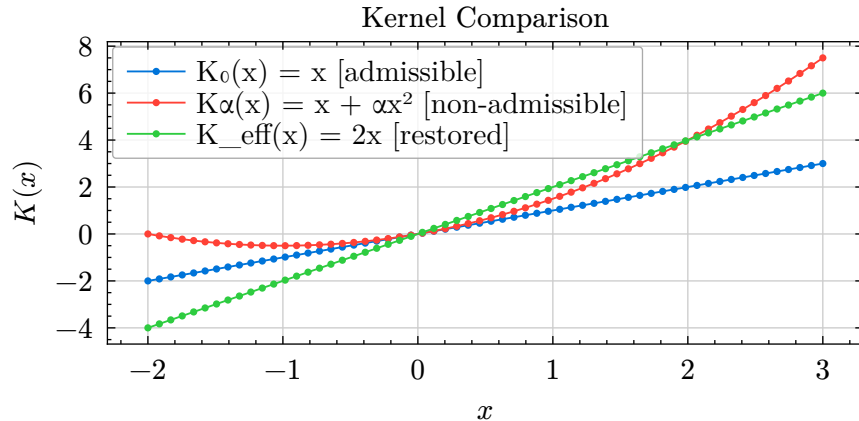


Figure 1: Three kernel types for  $\alpha = 0.5$ ,  $\varphi_0 = 1$ . Blue: admissible kernel  $K_0(x) = x$  (Class 1, linear). Red: non-admissible kernel  $K_\alpha(x) = x + 0.5x^2$  (curved — the quadratic term is the analog of the Yang–Mills self-coupling  $A \wedge A$ ). Green: restored kernel  $K_{\text{eff}}(x) = 2x$  (Class 2, linearized at  $\varphi_0 = 1$ ). The restored kernel is linear (admissible) but has a steeper slope — the slope difference  $m = 2\alpha\varphi_0 = 1$  is the characteristic scale (structural mass).

admissible kernel from the original non-admissible one plus the preferred value  $\varphi_0$ . This is the structural analog of what happens in gauge theory: the effective action for the gauge field after symmetry breaking is a genuinely different (massive, linear) theory, not a local approximation to the original (massless, non-linear) one. The defect cancellation is exact, not perturbative.

Figures 1–2 and Table 1 display these results numerically for  $\alpha = 0.5$  and  $\varphi_0 = 1$ .

### 7.1 Multi-Dimensional Extension: Directional Restoration

The scalar model above captures the algebraic mechanism of restoration. We now extend it to a three-dimensional Lie algebra, showing that the restoring field introduces **direction-dependent** mass: some gauge boson directions become massive while others remain massless.

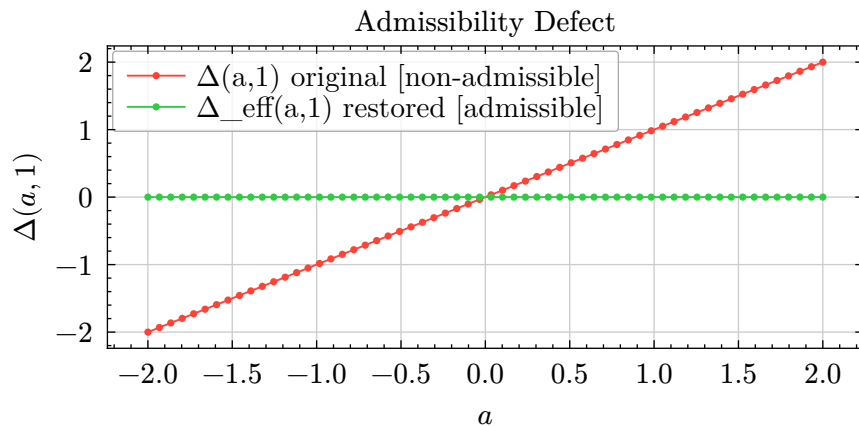


Figure 2: Admissibility defect  $\Delta(a, 1)$  as a function of input  $a$ , with  $b = 1$  fixed and  $\alpha = 0.5$ . Red: original kernel defect  $\Delta = 2\alpha ab = a$  (non-zero, confirming non-admissibility). Green: composite kernel defect  $\Delta_{\text{eff}} = 0$  (identically zero, confirming restoration). The vanishing of the composite defect is the constructive demonstration that restoration eliminates non-linearity.

$a$	$b$	$\Delta_{\text{orig}}$	$\Delta_{\text{eff}}$	$m = 2\alpha\varphi_0$
0.0	1.0	0.000	0.000	1.000
0.5	1.0	0.500	0.000	1.000
1.0	1.0	1.000	0.000	1.000
1.5	1.0	1.500	0.000	1.000
2.0	1.0	2.000	0.000	1.000
1.0	2.0	2.000	0.000	1.000

Table 1: Admissibility defect values for selected inputs ( $\alpha = 0.5$ ,  $\varphi_0 = 1$ ). The original kernel's defect grows as  $2\alpha ab$ , confirming non-admissibility. The composite (restored) kernel's defect is identically zero for all inputs. The mass scale  $m = 2\alpha\varphi_0 = 1.0$  emerges from the linearization.

**The model.** Consider a Lie algebra with three generators  $T_1, T_2, T_3$  satisfying  $[T_i, T_j] = \varepsilon_{ijk}T_k$  (the  $su(2)$  bracket). A gauge field  $A = a_1T_1 + a_2T_2 + a_3T_3$  has three components. The bracket  $[A, B]$  is non-zero for generic  $A, B$ : the algebra is non-abelian and the kernel is non-admissible.

**The restoring field.** Let  $\varphi_0 = v \cdot T_3$  — the restoring field picks the third direction in the Lie algebra. The **action** of the restoring field on each generator is the commutator  $[T_i, \varphi_0]$ :

$$[T_1, vT_3] = -vT_2 \neq 0, \quad [T_2, vT_3] = vT_1 \neq 0, \quad [T_3, vT_3] = 0$$

**Directional structure.** The mass acquired by a gauge boson in direction  $T_i$  is proportional to  $|[T_i, \varphi_0]|$ :

- $T_3$  (parallel to  $\varphi_0$ ): commutes with  $\varphi_0$ , mass = 0 — **unbroken, massless**.
- $T_1, T_2$  (perpendicular to  $\varphi_0$ ): do not commute with  $\varphi_0$ , mass =  $v$  — **broken, massive**.

For a general gauge boson direction at angle  $\psi$  to  $\varphi_0$ :

$$m(\psi) = v \cdot \sin(\psi)$$

The mass is zero along  $\varphi_0$  and maximal perpendicular to it. This is the structural origin of the directional pattern in gauge boson masses.

**The three classes within one algebra.** Remarkably, a single Lie algebra can contain all three classes of the classification simultaneously:

Direction	Relation to $\varphi_0$	Mass	Class
$T_3$	Parallel (commutes)	0	Admissible
$T_1, T_2$	Perpendicular	$v$	Restored
No $\varphi_0$	—	Confined	Unrestored

The three-class classification is not just a classification of different gauge groups — it is a classification of **directions within a single group**. The photon (massless) and  $W^\pm$  bosons (massive) live in the same  $SU(2) \times U(1)$  algebra, distinguished by their angle to the restoring field.

Figure 3 displays the angular dependence of the mass scale.

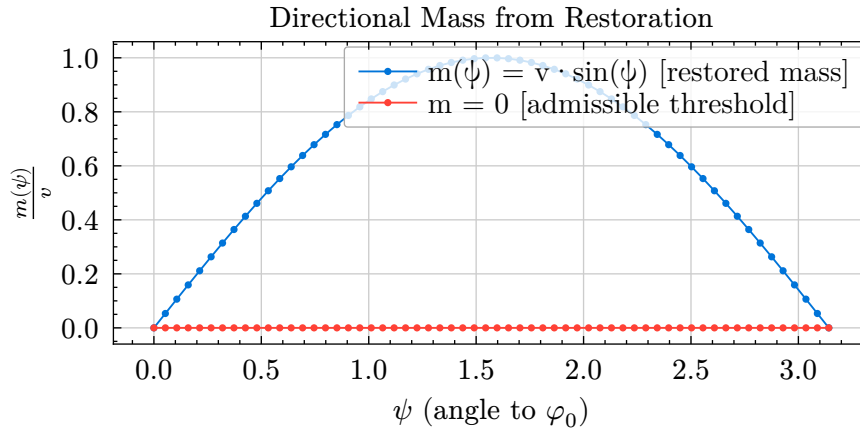


Figure 3: Mass scale  $m(\psi) = v \cdot \sin(\psi)$  as a function of the angle  $\psi$  between the gauge boson direction and the restoring field  $\varphi_0$  ( $v = 1$ ). At  $\psi = 0$  (parallel to  $\varphi_0$ ): mass = 0 (the unbroken direction, corresponding to the photon). At  $\psi = \pi/2$  (perpendicular to  $\varphi_0$ ): mass =  $v$  (maximally broken, corresponding to  $W^\pm$  bosons). The red line at  $m = 0$  marks the admissible (massless) threshold. The entire three-class classification is visible: massless at  $\psi = 0$ , massive at  $\psi > 0$ , and confined if no restoring field exists (not shown — all masses would be zero but charges would be unobservable).

## 8 Discussion

We address what this paper derives, what it does not derive, and its relation to prior work.

### 8.1 What We Derive

From the admissibility framework and the requirement that charges be observable:

1. **Restoration requires a field** (Theorem 1–2): if a non-abelian theory’s charges are observed, a restoring field must exist.
2. **The field must be gauge-charged** (Theorem 3): a gauge-invariant field cannot compensate a gauge-covariant defect.
3. **The field must have a preferred non-zero value** (Theorem 4): trivial fields cannot restore.
4. **Restoration introduces a scale** (Theorem 5): the preferred value defines a characteristic range.
5. **The three-class classification** (Theorem 6): all gauge theories are classified by admissibility and restoration.

These are structural results. They hold for any realization of gauge theory — quantum or classical, perturbative or non-perturbative, lattice or continuum.

### 8.2 What We Do Not Derive

This paper does not derive:

- The **spin** of the restoring field. Our constraints are satisfied by scalars (the Standard Model Higgs), fermion condensates (Technicolor), composite states (Composite Higgs), and extra-dimensional gauge components (gauge-Higgs unification). The algebraic framework does not select among these.

- The **representation** of the restoring field. We derive that it must be gauge-charged, but not whether it transforms in the fundamental, adjoint, or other representation.
- The **potential** of the restoring field. The Mexican hat’ potential  $V(\varphi) = -\mu^2 |\varphi|^2 + \lambda |\varphi|^4$  is a dynamical assumption. We derive the existence of a preferred value, not the mechanism that selects it.
- The **mass** of the physical Higgs boson (125 GeV) or the gauge bosons ( $M_W \approx 80$  GeV,  $M_Z \approx 91$  GeV). These are dynamical quantities.
- The **breaking pattern**  $SU(2) \times U(1) \rightarrow U(1)_{\text{em}}$ . We derive that the restoring field breaks the gauge symmetry, but not which subgroup survives.
- The **Yukawa couplings** that generate fermion masses.

We also do not claim that admissibility restoration is the **unique** mechanism by which a non-abelian gauge theory could have observable charges. Our framework formalizes one mechanism — the introduction of a restoring field that compensates the defect — and derives its consequences. Whether other mechanisms exist that make charges observable without restoring kernel admissibility is an open question. The conservative position is: within the POT framework, restoration is the only mechanism we have formalized; we do not claim to have proven that no alternative exists. The three-class classification is exhaustive **given** the admissibility framework; it is not claimed to be exhaustive over all conceivable mechanisms.

These limitations are honest. The paper contributes a **structural necessary condition** for restoration, not a complete theory of electroweak symmetry breaking. It is an analytical result — derived from algebraic structure, not from experiment. We have not performed laboratory experiments or observational measurements; we have identified structural constraints that any empirical theory of admissibility restoration must satisfy.

### 8.3 The Higgs as Skolem Witness

The structural result has the logical form of an existential statement:

$$\text{Charges observable} \wedge \text{kernel non-admissible} \Rightarrow \exists \varphi. \text{restores\_admissibility}(\varphi, K)$$

In model theory, a **Skolem witness** is a specific element satisfying an existential claim. The Standard Model Higgs field — a complex scalar doublet with hypercharge  $Y = 1/2$  and vacuum expectation value  $v \approx 246$  GeV — is a specific Skolem witness for this structural requirement. It satisfies all three constraints:

1. **Gauge-charged:** the Higgs doublet transforms under  $SU(2)_L \times U(1)_Y$ .
2. **Preferred non-zero value:** the vacuum expectation value  $v = (\sqrt{2}G_F)^{-1/2} \approx 246$  GeV.
3. **Introduces a scale:**  $M_W = gv/2 \approx 80$  GeV,  $M_Z = M_W/\cos\theta_W \approx 91$  GeV.

But the Skolem witness is not unique. Technicolor models (Weinberg, 1976; Susskind, 1979) provide an alternative witness: a fermion condensate  $\langle \bar{\psi}\psi \rangle \neq 0$  that is gauge-charged and has a preferred non-zero value. Composite Higgs models and gauge-Higgs unification provide further witnesses. The structural theorem requires that a witness exist; it does not determine **which** witness nature selects. That determination is empirical — and the LHC has identified the Standard Model scalar as the physical realization.

## 8.4 Fradkin–Shenker Complementarity

A result of Fradkin and Shenker (1979) poses an important question for our classification. When the Higgs field transforms in the fundamental representation of the gauge group, the Higgs phase and the confinement phase are **smoothly connected**: there is no phase boundary separating them. One can continuously interpolate between the weak-coupling (Higgs) regime and the strong-coupling (confinement) regime.

‘t Hooft reinterpreted this as follows: the Higgs mechanism **is** confinement, in a different regime. The  $W$  and  $Z$  bosons are gauge-invariant composite operators (bound states of elementary fields), just as hadrons are gauge-invariant composites of quarks and gluons. The difference is quantitative — the binding’ scale — not qualitative.

This is consistent with our framework. Both Class 2 (restored) and Class 3 (unrestored) kernels are non-admissible. The Lie bracket  $[A, B]$  is present in both cases. The structural difference is not in the kernel itself but in the **existence of a restoring field**: in the weak sector, a field  $\varphi$  with  $\varphi_0 \neq 0$  compensates the defect at the scale  $|\varphi_0|$ ; in the strong sector, no such field exists, and the defect operates at all scales.

The Fradkin–Shenker continuity suggests that the boundary between Class 2 and Class 3 may be porous: by varying parameters, one could in principle move from the restored to the unrestored regime. In the POT framework, this corresponds to the question of whether  $\varphi_0$  can be continuously reduced to zero — whether the preferred value can melt.’ At  $\varphi_0 = 0$ , restoration fails and confinement is recovered. This is the structural analogue of the deconfinement phase transition: the restoring field’s preferred value is the order parameter.

We note this connection but do not formalize it further. The Fradkin–Shenker result operates within the quantized theory; our framework is pre-quantum. The compatibility between the two is a consistency check, not a derivation.

## 8.5 Relation to Prior Work

Several approaches have addressed the question of **why** a Higgs-like field must exist:

- **Unitarity bounds** (Lee, Quigg, and Thacker, 1977): Without the Higgs, longitudinal  $WW$  scattering violates unitarity at  $\sim 1$  TeV. This is a dynamical consistency argument within perturbative QFT. Our argument is pre-quantum and does not require scattering amplitudes.
- **Renormalizability** (‘t Hooft, 1971): Gauge theories with spontaneous symmetry breaking are renormalizable. This established the theoretical viability of the electroweak theory but does not explain **why** the breaking must occur. Our result provides a structural reason: without restoration, charges would be confined.
- **Elitzur’s theorem** (1975): Local gauge symmetry cannot be spontaneously broken. This is consistent with our framework: we do not invoke symmetry breaking. Admissibility restoration is a distinct concept — it concerns the defect of the composite kernel, not the symmetry of the vacuum.

- The **Frohlich–Morchio–Strocchi (FMS) mechanism** (1980–81): Physical particles in the electroweak theory are gauge-invariant composites, not elementary fields. The FMS mechanism is the closest algebraic predecessor to our approach. Both frameworks emphasize gauge-invariant observables and avoid the language of symmetry breaking. The difference is that FMS operates within algebraic QFT; our result is pre-quantum.
- **Gauge-Higgs unification** (Hosotani, 1983): The Higgs is identified as an extra-dimensional gauge component. This is a structural derivation but requires extra dimensions. Our derivation assumes nothing about spacetime dimensionality.
- **Anderson’s plasmon mechanism** (1963): In condensed matter, the Nambu–Goldstone mode combines with the gauge field to produce a massive vector boson. This is the physical precursor to the Higgs mechanism. Anderson assumed the existence of the order parameter (the superconducting condensate); we derive the necessity of the restoring field from the kernel algebra.

The unique contribution of the present work is that the necessity argument operates **before** quantization. All prior arguments require the apparatus of quantum field theory — scattering amplitudes, renormalization, the Hilbert space, or at minimum a quantum vacuum. Our argument requires only the admissibility of the kernel and the observability of charges.

## 8.6 The Observability Principle is a Theorem

A natural objection to the entire POT framework is that the link between fiber invariance and observability is assumed, not derived. If observability requires a separate postulate beyond the projection structure, then the framework rests on an independent physical assumption that could be challenged.

We show this objection is unfounded. The fiber observability principle is not an axiom — it is a **theorem** of the projection structure.

**Definition.** Let  $P : X \rightarrow Y$  be a projection from the ontological space  $X$  to the observable space  $Y$ . The fiber over  $y \in Y$  is  $P^{-1}(y) = \{x \in X : P(x) = y\}$ . A predicate  $Q$  on  $X$  is **fiber-invariant** if  $Q(x) = Q(x')$  whenever  $P(x) = P(x')$ .

**Definition.** A predicate  $Q$  is **observable** if its value can be determined from the projection image  $P(x)$  alone — that is, if  $Q(x) = Q(x')$  whenever  $P(x) = P(x')$ .

**Theorem 7 (Fiber Observability).** A predicate is observable if and only if it is fiber-invariant.

**Proof.** The two definitions have identical logical content: both require  $P(x) = P(x') \Rightarrow Q(x) = Q(x')$ . The equivalence is a tautology of the projection structure.  $\square$

**Corollary.** Non-invariant predicates are unobservable.

The triviality of the proof is the point. The axiom<sup>4</sup> that Volumes I–V relied upon was never an independent assumption — it was always a logical consequence of what projection<sup>4</sup> means. The only genuine axiom of POT is:

*Observables are images of a projection.*

This is the claim that observation involves information loss — that we see a reduced description of what exists. This claim is not controversial: thermodynamics hides microstates, quantum mechanics hides wavefunctions, gauge theory hides gauge orbits, general relativity hides coordinate dependence. In every case, the observable is a projection of something richer. Rejecting this axiom means asserting that observation is lossless — that we see reality exactly as it is. No physical theory has ever made this assertion.

**Connection to gauge invariance.** In gauge theory, the projection  $P$  is the quotient map from field configurations to gauge orbits:  $P(A) = [A]$ . The fiber is the gauge orbit. A gauge transformation  $g$  maps  $A$  to  $A^g$  within the same orbit, so  $P(A) = P(A^g)$ . Fiber invariance therefore implies gauge invariance. Elitzur’s theorem — gauge-variant operators have vanishing expectation value — is a special case of the fiber observability theorem.

All seven results (reflexivity, symmetry, and transitivity of fiber equivalence; the equivalence theorem; the contrapositive; the gauge connection; and the stress test that no observable distinguishes fiber-equivalent states) are verified by Z3 in the theory file `pot_fiber_observability.kleis`.

## 9 Falsifiability

The three-class classification makes falsifiable structural predictions:

**Prediction 1.** Any restoring field must be gauge-charged. If a gauge-singlet field were discovered to restore admissibility — making a non-abelian gauge boson massive without coupling to the gauge group — the structural theorem would be falsified.

**Prediction 2.** Any restoring field must have a preferred non-zero value. If massive gauge bosons were found in a non-abelian theory without any field acquiring a non-zero preferred value, the theorem would be falsified.

**Prediction 3.** The admissible sector ( $U(1)$ ) has a massless gauge boson. If the photon were discovered to have a non-zero mass in the absence of any restoring field, the classification would be falsified. Current experimental bounds ( $m_\gamma < 10^{-18}$  eV) are consistent.

**Prediction 4.** The unrestored sector ( $SU(3)$ ) must confine. If free quarks (isolated color charges) were observed in the absence of any admissibility-restoring field, the classification would be falsified.

**Prediction 5.** Every non-abelian gauge theory with observable charges must have a restoring field. If a new non-abelian gauge interaction were discovered with observable charges but no associated restoring field, the classification would require revision.

All five predictions are consistent with all known physics.

## 10 Conclusion

The weak interaction presents an apparent paradox for the structural confinement theorem:  $SU(2)$  is non-abelian, yet weak charges are observed. The resolution is admissibility restoration: a

field  $\varphi$  that couples to the non-admissible kernel and compensates its Lie bracket defect, making the composite kernel effectively admissible.

From this single concept, three structural constraints on the restoring field follow: it must be gauge-charged, it must have a preferred non-zero value, and it must introduce a characteristic scale. These are the defining properties of the Standard Model Higgs field — derived here as structural necessities, not postulated as dynamical assumptions.

The result yields a three-class classification of gauge theories:

1. **Admissible:** charges observable, gauge boson massless (electromagnetism).
2. **Non-admissible, restored:** charges observable, gauge boson massive (weak interaction).
3. **Non-admissible, unrestored:** charges confined (strong interaction).

This classification unifies the three gauge interactions of the Standard Model as three states of a single structural variable — kernel admissibility — without writing a Lagrangian, evaluating a path integral, or invoking any quantum-mechanical postulate. The dynamics that select which class each interaction belongs to — and that determine the specific properties of the restoring field (its spin, representation, potential, and mass) — require additional content beyond the algebraic framework formalized here.

The Standard Model Higgs is a Skolem witness for the existential requirement of admissibility restoration. That nature selected a fundamental scalar rather than a fermion condensate or an extra-dimensional gauge component is an empirical fact, not a structural necessity. The structure requires a witness; the dynamics select which one.

The theory file `pot_admissibility_restoration.kleis` is open source, machine-checkable, and extensible. The reader is invited to `kleis test it`.

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# Appendix

## A Complete Axiom Inventory

The formal theory `pot_admissibility_restoration.kleis` imports `pot_admissible_kernels_v2.kleis` and defines 9 structures:

1. **FieldNegation** — negation on the field algebra (2 axioms).
2. **CompositeDefect** — defect of the composite kernel (2 axioms).
3. **AdmissibilityRestoration** — definition of restoration (2 axioms).
4. **RestorationDichotomy** — three-way classification by admissibility and restoration (4 axioms).
5. **RestoringFieldConstraints** — gauge-charged, preferred value, zero element (4 axioms, 1 element).
6. **MassConsequence** — restoration introduces a scale (3 axioms).
7. **CompleteClassification** — exhaustive three-class assignment (3 axioms).

Additionally, the imported `pot_admissible_kernels_v2.kleis` contributes 5 structures with 14 axioms.

Total: 12 structures (7 new + 5 imported), 34 axioms, 16 Z3-verified examples.

All examples pass Z3 verification.