Projection Singularities: Why Physics Has No Infinities

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Abstract

Every physical infinity on record — ultraviolet divergences in quantum field theory, curvature singularities in general relativity — shares a common algebraic structure: it is a term in a formal power series that lies outside the admissible domain of the representation being used. We formalize this observation as the **projection singularity principle**: given any formal series expansion of a physical quantity, the Hadamard-product projection (a coefficient-wise filter that zeros out singular-index terms and preserves admissible-index terms) extracts exactly the physical observable. The image of this projection is the space of finite, measurable quantities. The null space is the space of infinities. We show that Hadamard's finite-part operator (used in QFT renormalization since 1948) and the resolution of coordinate singularities in GR are both instances of this single algebraic operation applied to different formal series. The projection is idempotent ($P^2 = P$), linear, and decomposes every series into an observable part and an artifact. Volume VII of this series proved that the QFT composite kernel $K_{\mathrm{QFT}} = \mathrm{FP} \circ K_{\mathrm{ren}} \circ K_{\mathrm{path}}$ maps sources to finite observables via the ITCM; we prove that the ITCM result is a special case of the Hadamard projection applied to the factored series representation. For general relativity, the Kretschner scalar $K = 48M^2/r^6$ of the Schwarzschild solution is shown to be entirely in the null space of the Hadamard projection when expanded in powers of $1/r$, making the curvature singularity a representational artifact rather than a physical feature. As a decisive worked example, we apply the projection end-to-end to the Passarino–Veltman scalar tadpole $A_0(m^2)$, a genuine one-loop QFT integral, and recover the textbook MS-renormalized value with no subtraction procedure. The ontological conclusion: physical observables correspond to elements stable under the admissibility projection; infinities are null-space elements of particular representations. There is nothing to renormalize and no singularity to resolve — the representation has a boundary; the physics does not. All 29 structural results are machine-verified by the Z3 SMT solver, and the worked example is verified by direct computation, in the Kleis formal verification language.

Keywords: projection singularity, Hadamard product, formal power series, renormalization, curvature singularity, finite part operator, admissibility domain, projection kernel, general relativity, quantum field theory, projected ontology, formal verification, Z3

1 Introduction

Physics has two infinity problems. In quantum field theory, loop integrals diverge: the perturbative expansion of scattering amplitudes contains terms proportional to $1/\varepsilon^n$ that blow up as the regularization parameter $\varepsilon \to 0$ [tH1972, BG1972]. In general relativity, curvature invariants diverge: the Kretschner scalar of the Schwarzschild solution goes as r^{-6} and becomes infinite at

$r = 0$ [Pen1965, HP1970]. These two problems are treated by two different communities using two different formalisms — renormalization for QFT [PS1995], singularity theorems for GR [HE1973] — with no recognition that they might be the same phenomenon.

This paper argues that they are.

Both infinities arise from the same algebraic mechanism: a formal power series is being evaluated outside the domain where the series faithfully represents the physics. The QFT divergence comes from the Laurent expansion of regularized amplitudes at negative powers of ε. The GR divergence comes from the expansion of curvature invariants at positive powers of $1/r$. In both cases, the infinity lives in a specific set of coefficients — the **singular indices** — while the physical content lives in the complementary set — the **admissible indices**.

The tool that separates them is the Hadamard-product projection: a coefficient-wise filter P that preserves admissible-index coefficients and zeros out singular-index coefficients. This projection is:

- **Idempotent**: $P^2 = P$ (it is genuinely a projection, not a procedure)
- **Linear**: $P(f + g) = P(f) + P(g)$ (it respects superposition)
- **Domain-adapted**: what counts as 'admissible' depends on the representation, not the physics

The Mercator projection provides the intuitive analogy. It maps the sphere faithfully to the plane everywhere except at the poles, where the scale factor diverges. The infinity is in the map, not the earth. The Hadamard projection is the formal version of 'discard the terms that come from pushing the map beyond its domain.'

This paper is Volume X of the Projected Ontology Theory (POT) series. Volume VII established that renormalization is a projection kernel and that the composite QFT kernel $K_{\mathrm{QFT}} = \mathrm{FP} \circ K_{\mathrm{ren}} \circ K_{\mathrm{path}}$ has a closed-form representation via the Integral Transform Composition Method (ITCM). The present paper subsumes that result: the ITCM composite kernel is what you get when you apply the Hadamard projection to the factored series and bypass the divergent intermediate entirely.

The paper is organized as follows. Section 2 defines formal power series and the admissibility domain. Section 3 constructs the Hadamard projection and proves its algebraic properties. Section 4 establishes the image/nullspace decomposition. Section 5 applies the framework to QFT renormalization. Section 6 applies it to GR curvature singularities. Section 7 formalizes the Mercator analogy as the projection singularity principle. Section 8 proves universality: QFT and GR are instances of the same operation. Section 9 connects to the ITCM composite kernel of Volume VII. Section 10 draws the ontological conclusion.

2 Formal Power Series and the Admissibility Domain

Every divergent quantity in physics admits a formal series expansion in some parameter. For QFT amplitudes, this is the Laurent series in ε (dimensional regularization [tH1972, BG1972]) or Λ (cutoff). For GR curvature invariants, this is the series in $1/r$ (radial expansion). For spectral sums, this is the Dirichlet series in s (zeta regularization). Dyson [Dys1952] showed that perturbation series in QED are asymptotic rather than convergent, making the formal-series viewpoint not merely convenient but necessary.

We abstract this pattern. A **formal series** $f = \sum_n a_n x^n$ assigns a coefficient a_n to each integer index $n \in \mathbb{Z}$. The index set $\mathbb{Z}$ (rather than $\mathbb{N}$) allows Laurent series with negative-power terms.

The **admissibility domain** is a partition of the index set into two classes:

$$A = \{n \in \mathbb{Z} : n \text{ is admissible}\}, \quad S = \{n \in \mathbb{Z} : n \text{ is singular}\}$$

such that $A \cup S = \mathbb{Z}$ and $A \cap S = \emptyset$.

For QFT: $A = \{n \geq 0\}$, $S = \{n < 0\}$. The admissible terms are the constant and positive powers of ε; the singular terms are the poles $1/\varepsilon, 1/\varepsilon^2, \ldots$.

For GR (in powers of $u = 1/r$): $A = \{n \leq 0\}$, $S = \{n > 0\}$. The admissible terms are those that remain bounded as $r \to 0$; the singular terms diverge.

The crucial observation is that the partition is a property of the **representation** (the choice of expansion parameter), not the **physics**. The same physical quantity can be expanded in different parameters, each producing a different partition. This is the formal content of 'the infinity is in the map, not the territory.'

The admissibility domain is not freely chosen. It is **constrained** by the requirement that admissible terms remain bounded, measurable, and stable under the observable limit (the $\varepsilon \to 0$ or $r \to 0$ evaluation). In QFT, the admissible indices $n \geq 0$ are exactly those whose contributions $a_n \varepsilon^n$ remain finite as $\varepsilon \to 0$. In GR, the admissible indices are those whose contributions do not diverge as $r \to 0$. The partition is determined by the analytic structure of the expansion, not declared by convention.

All structures in this section are verified by Z3 in `theories/pot_projection_singularity.kleis`, Parts 1–2.

3 The Hadamard Projection

Given a formal series f and an admissibility domain A, the **Hadamard projection** P is defined coefficient-wise:

$$[P(f)]_n = \begin{cases} a_n & \text{if } n \in A \\ 0 & \text{if } n \in S \end{cases}$$

Equivalently, $P(f) = f \odot \mathbb{1}_A$, the Hadamard product of f with the indicator series of the admissible domain.

Theorem 1 (Projection properties). P satisfies:

1. **Idempotency**: $P(P(f)) = P(f)$ for all f.
2. **Linearity**: $P(f + g) = P(f) + P(g)$ for all f, g.
3. **Kill property**: If $n \in S$, then $[P(f)]_n = 0$.
4. **Preserve property**: If $n \in A$, then $[P(f)]_n = a_n$.
5. **Zero preservation**: $P(0) = 0$.

Proof. Each property follows directly from the coefficient-wise definition. Idempotency: $[P(P(f))]_n = [P(f)]_n$ because $P(f)$ already has zero coefficients at singular indices. Linearity:

$[P(f+g)]_n = [f+g]_n = a_n + b_n = [P(f)]_n + [P(g)]_n$ when $n \in A$, and $0 + 0 = 0$ when $n \in S$. The remaining properties are immediate. □

Theorem 1b (Uniqueness). Given a partition $\mathbb{Z} = A \sqcup S$, the Hadamard projection P is the **unique** linear idempotent satisfying the kill and preserve properties.

Proof. Linearity and the two properties determine P on a basis. Every formal series decomposes as $f = f_A + f_S$ where f_A is supported on A and f_S on S. Then $P(f) = P(f_A) + P(f_S) = f_A + 0 = f_A$. This is exactly the coefficient-wise filter. Any other linear idempotent satisfying kill and preserve must agree with P on every basis element, hence on every series. □

The significance: given the admissibility partition, there is no freedom in the projection. The only choice is the partition itself — and the partition is forced by boundedness and measurability (Section 2). The projection is not one option among many. It is the unique operator that respects the analytic structure of the representation.

These properties are axiomatized in `HadamardProjection` and verified by Z3 in Part 3 of the theory file.

4 Image and Nullspace

The Hadamard projection decomposes the space of formal series into a direct sum:

$$\text{FormalSeries} = \text{Image}(P) \oplus \text{Null}(P)$$

where:

- $\text{Image}(P) = \{f : P(f) = f\}$ — the **admissible series**, with zero coefficients at all singular indices.
- $\text{Null}(P) = \{f : P(f) = 0\}$ — the **purely singular series**, with zero coefficients at all admissible indices.

Every series decomposes uniquely: $f = P(f) + (f - P(f))$, where $P(f) \in \text{Image}(P)$ and $(f - P(f)) \in \text{Null}(P)$.

This is precisely the POT kernel decomposition established in Volumes I–III:

- $\text{Image}(K)$ = physical observables
- $\text{Null}(K)$ = gauge/artifact degrees of freedom

The Hadamard projection is therefore an admissible kernel in the POT sense. Its image is the space of finite, measurable quantities. Its null space is the space of infinities. The choice of admissibility domain (which partition of indices to use) is the gauge freedom.

Theorem 2. For every formal series f, the projected series $P(f)$ is in the image: $P(P(f)) = P(f)$.

Proof. Immediate from idempotency (Theorem 1.1). □

Theorem 3. The zero series is in the null space: $P(0) = 0$.

Proof. Immediate from zero preservation (Theorem 1.5). □

Both theorems are verified by Z3 in Part 4.

5 QFT: Renormalization as Hadamard Projection

The standard approach to UV divergences proceeds by subtraction: the BPHZ forest formula recursively subtracts divergent subgraphs [Sch1948, tH1972], while the Connes–Kreimer Hopf algebra [CK2000] reveals the algebraic structure underlying this recursion. An alternative tradition, initiated by Epstein and Glaser [EG1973], avoids divergences entirely by constructing the S-matrix inductively via distribution splitting, so that ill-defined products of distributions never arise. Our approach is closest in spirit to Epstein–Glaser: we do not subtract divergences, but identify the admissible content directly via projection.

Volume VII of this series established that renormalization is a projection kernel: the finite part operator FP extracts the constant term from the Laurent expansion of a regularized value, the singular part (poles) lives in the null space, and the choice of regularization scheme is gauge freedom.

We now identify the finite part operator with the Hadamard projection.

Every regularized value $F(\varepsilon)$ has a Laurent expansion:

$$F(\varepsilon) = \sum_{n=-N}^{\infty} a_n \varepsilon^n$$

The finite part is the constant term: $\mathrm{FP}(F) = a_0$. But this is precisely what the Hadamard projection gives when the admissibility domain is $A = \{n \geq 0\}$:

$$P(F) = \sum_{n=0}^{\infty} a_n \varepsilon^n$$

and evaluation at $\varepsilon = 0$ yields $a_0 = \mathrm{FP}(F)$.

Theorem 4 (FP is Hadamard). For every regularized value F:

$$\mathrm{FP}(F) = [P(\text{series\_of}(F))]_{n=0}$$

Proof. The Laurent series of F has coefficients a_n for $n \in \mathbb{Z}$. The Hadamard projection with $A = \{n \geq 0\}$ zeros out a_n for $n < 0$. Evaluation at $\varepsilon = 0$ kills all a_n for $n > 0$, leaving a_0. This is the definition of $\mathrm{FP}(F)$. □

Theorem 5 (Gauge invariance via projection). If two regulators R_1, R_2 are gauge-equivalent (they yield the same finite part for all divergent objects), then they produce the same Hadamard projection:

$$R_1 \sim R_2 \quad \Rightarrow \quad P(\text{series\_of}(R_1(X))) = P(\text{series\_of}(R_2(X)))$$

Proof. Gauge-equivalent regulators differ only in the singular part of their Laurent expansions. The Hadamard projection annihilates the singular part, leaving only the admissible part, which is identical by definition of gauge equivalence. □

Both theorems are axiomatized in `QFTHadamardBridge` and verified by Z3 in Part 5.

5.0.1 Worked example: the scalar tadpole $A_0(m^2)$

As a concrete end-to-end demonstration, we apply the Hadamard projection to the Passarino–Veltman scalar one-point function $A_0(m^2)$ [PV1979, tHV1979], the simplest nontrivial loop integral in QFT:

$$A_0(m^2) = \mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \cdot \frac{1}{k^2 + m^2}, \quad d = 4 - 2\varepsilon$$

In dimensional regularization [tH1972, BG1972], this evaluates to [PS1995, §10.2]:

$$A_0(m^2) = \frac{m^2}{16\pi^2} \left[\frac{1}{\varepsilon} + \left(1 - \gamma_E + \ln\left(4\pi \frac{\mu^2}{m^2} \right) \right) + O(\varepsilon) \right]$$

The Laurent series has two coefficients: $a_{-1} = \frac{m^2}{16\pi^2}$ (the UV pole) and $a_0 = \frac{m^2}{16\pi^2} \cdot \left(1 - \gamma_E + \ln\left(4\pi \frac{\mu^2}{m^2}\right)\right)$ (the finite part). The Hadamard projection with $A = \{n \geq 0\}$ yields:

$$P(A_0) = [0, a_0]$$

At $\mu = m = 1$: $a_{-1} \approx 0.006333$, $a_0 \approx 0.018705$. The projection annihilates the pole and preserves the finite part. The result matches the MS-renormalized textbook value exactly. No subtraction was performed; the projection identified the admissible content and discarded the rest.

Full computation with all intermediate values, idempotency, decomposition, and nullspace verification is in `theories/pot_projection_decisive_a0.kleis` (7 machine-verified examples, all passing).

5.0.2 Structure test: the scalar bubble $B_0(p^2; m^2, m^2)$

The tadpole A_0 has a constant finite part. A stronger test: does the projection preserve nontrivial kinematic structure? The Passarino–Veltman scalar two-point function $B_0(p^2; m^2, m^2)$ [PV1979, tHV1979] has a finite part that depends on external momentum through a Feynman parameter integral:

$$B_0 = \frac{1}{\varepsilon} - \gamma_E + \ln\left(4\pi \frac{\mu^2}{m^2} \right) - \int_0^1 dx \ln(1 + \rho \cdot x(1 - x)) + O(\varepsilon)$$

where $\rho = |p^2 \frac{|}{m^2}$. The UV pole $a_{-1} = 1$ is the same at all kinematic points. The finite part a_0 varies: at $\rho = 1$ (spacelike $p^2 = -m^2$), $a_0 \approx 1.802$; at $\rho = 4$ ($p^2 = -4m^2$), $a_0 \approx 1.460$.

The Hadamard projection at both points annihilates the identical pole and preserves the different finite parts. The difference in observables $\Delta a_0 = a_0(\rho = 1) - a_0(\rho = 4) \approx 0.341$ equals exactly the difference in Feynman parameter integrals. The projection preserves the momentum-dependent logarithmic structure of the amplitude.

Full computation at both kinematic points, including structure preservation, nullspace universality, decomposition, and linearity, is in `theories/pot_projection_decisive_b0.kleis` (6 machine-verified examples, all passing).

6 GR: Curvature Singularities as Projection Artifacts

The Schwarzschild metric describes the spacetime geometry of a non-rotating, uncharged black hole. Its curvature is characterized by the Kretschner scalar:

$$K = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{48M^2}{r^6}$$

This diverges as $r \to 0$. The standard interpretation is that spacetime is 'infinitely curved' at the center of the black hole — a genuine physical singularity that signals the breakdown of GR.

We offer a different reading. Expand the Kretschner scalar in powers of $u = 1/r$:

$$K(u) = 48M^2u^6$$

This is a monomial in u with all its weight at index $n = 6$. The admissibility domain for physical curvature observables (quantities that remain bounded as $r \to 0$, i.e., as $u \to \infty$) is $A = \{n \leq 0\}$ in powers of u.

Since $n = 6 > 0$, the index is singular. The Hadamard projection yields:

$$P(K) = 0$$

The Kretschner scalar is **entirely in the null space** of the Hadamard projection. Under the admissibility projection defined by boundedness of the observable representation, the curvature divergence lies outside the observable image.

Note carefully what this does and does not claim. The Kretschner scalar is a scalar invariant: it is independent of coordinates. The claim is **not** that the divergence is a coordinate artifact in the classical GR sense. The claim is stronger and more specific: the divergence lies outside the admissible domain of the **power-series representation** in which the curvature is expressed. A different representation (a different series parameter, a different kernel) may render the same physical content as a bounded quantity. The infinity is diagnostic: it tells you where your representation breaks down, not where the physics does.

Theorem 6. For any curvature invariant C: if C is unbounded at the origin, then $P(\text{curvature\_series}(C)) = 0$.

Proof. If C is unbounded, all non-zero coefficients of its series expansion are at singular indices. The Hadamard projection zeros them out. □

Corollary. The Kretschner scalar of the Schwarzschild solution projects to zero: $P(K) = 0$.

6.0.1 Relation to the Penrose singularity theorem

The Penrose singularity theorem [Pen1965], generalized by Hawking and Penrose [HP1970], proves that under physically reasonable conditions (energy conditions, existence of a trapped surface, global hyperbolicity), spacetime is **geodesically incomplete**: there exist causal geodesics that cannot be extended to infinite affine parameter. This is often paraphrased as 'singularities must exist,' but the theorem's content is more precise: it proves that the classical GR description cannot be extended globally, not that physically real infinities must appear.

Our framework is compatible with Penrose's result. Geodesic incompleteness identifies the **boundary** of the classical representation; the Hadamard projection identifies which parts of that representation correspond to physical observables. In the language of this paper: Penrose proves that the representation has a boundary. We prove that the divergences arising at that boundary lie in the null space of the admissibility projection. The two results are complementary, not contradictory.

Both are verified by Z3 in Part 6 of the theory file.

7 The Projection Singularity Principle

The Mercator projection provides the template. It maps the sphere to the plane via:

$$x = \lambda, \quad y = \ln\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)$$

where λ is longitude and φ is latitude. As $\varphi \to \pm 90°$, the y-coordinate diverges. The poles are sent to infinity. But the poles have not become infinite — they are finite patches of the earth with well-defined area and curvature. The infinity is in the **representation** (the Mercator projection), not the **territory** (the sphere).

We formalize this as the **projection singularity principle**:

Definition. A **projection singularity** is a divergence in a formal series representation of a physical quantity that vanishes under the Hadamard projection. Formally: a representation R has a projection singularity if its series $\text{series}(R)$ is not in the image of P, but $P(\text{series}(R))$ is.

Theorem 7 (Faithful representations are in the image). If a representation R is faithful (no divergences), then $\text{series}(R) \in \text{Image}(P)$.

Theorem 8 (Projection always recovers a faithful part). For any representation R, the Hadamard projection $P(\text{series}(R))$ is in the image of P.

Proof of Theorem 8. This is Theorem 2 applied to $\text{series}(R)$: $P(P(\text{series}(R))) = P(\text{series}(R))$. □

The projection singularity principle asserts: **any singularity that vanishes under Hadamard projection is a property of the representation, not the physics.** The Mercator poles, QFT UV divergences, and GR curvature singularities are all projection singularities.

Verified by Z3 in Part 7.

8 Universality: Same Operation, Two Domains

QFT and GR divergences are structurally identical through the Hadamard projection. The following table summarizes the correspondence:

The claim is not that QFT and GR are analogous. It is that they are **instances of the same mathematical operation** — the Hadamard projection — applied to different formal series arising from different physical contexts.

Theorem 9. For every physical domain D, the observable is the Hadamard projection of the domain's series:

	QFT (Renormalization)	**GR (Singularities)**
Source	Path integral output	Curvature invariant
Series parameter	ε (dimensional reg.)	$1/r$ (radial expansion)
Admissible	$n \geq 0$	$n \leq 0$ in $1/r$
Singular	$n < 0$ (poles)	$n > 0$ (divergences)
Projection	FP = Hadamard project	Hadamard project
Image	Finite observable	Bounded curvature
Nullspace	UV divergent terms	Curvature singularity
Gauge freedom	Regularization scheme	Coordinate chart

Table 1: Universality of the Hadamard projection across QFT and GR.

$$\text{Observable}(D) = P(\text{series}(D))$$

Theorem 10. If a domain has a projection singularity, its raw series is not in the image of P:

$$\mathsf{has\_projection\_singularity}(D) \Rightarrow \mathsf{series}(D) \notin \mathsf{Image}(P)$$

Both QFT and GR have projection singularities. Both produce finite observables via P. The universality is not metaphorical — it is algebraic.

Verified by Z3 in Part 8.

9 Connection to the ITCM Composite Kernel

Volume VII proved that the composite QFT kernel

$$K_{\text{QFT}} = \text{FP} \circ K_{\text{ren}} \circ K_{\text{path}}$$

maps sources directly to finite observables, and that the Integral Transform Composition Method (ITCM) of Sitnik and collaborators provides a closed-form hypergeometric representation for the composed operator.

The Hadamard projection subsumes this result. The factored computation $K_{\text{path}} \to K_{\text{ren}} \to$ FP passes through a divergent intermediate (the output of K_{path} before regularization). But the composite kernel, viewed as a single operator, never produces a divergent intermediate. Its codomain is $\mathsf{FinitePart}$, not $\mathsf{RegValue}$.

In the language of the Hadamard projection:

$$K_{\text{QFT}}(J) = \text{FP}\big[K_{\text{ren}}\big(K_{\text{path}}(J)\big)\big] = P\big(\mathsf{series\_of}\big(K_{\text{ren}}\big(K_{\text{path}}(J)\big)\big)\big)|_{\varepsilon=0}$$

The ‘divergences were never real’ result of Volume VII (Part 11) is a special case of the projection singularity principle: the divergences are in the **factored representation** $K_{\text{path}}, K_{\text{ren}}, \text{FP}$, not in the composed operator K_{QFT}. The ITCM hypergeometric kernel computes the output of the Hadamard projection directly, without passing through the singular intermediate.

Theorem 11. The direct composite equals the Hadamard projection of the factored series:

$$K_{\mathrm{QFT}}(J) = P(\text{composite\_series}(J))$$

Theorem 12. The factored series has projection singularities (it is not in the image of P), but the Hadamard projection of the factored series is.

The pattern is universal: factor an operator into steps, and the intermediate may diverge. Compose the factors into a single operator, and the divergence disappears. The infinity was in the factorization, not the physics.

Verified by Z3 in Part 9.

10 The Ontological Conclusion

The projection singularity principle leads to a sharp ontological claim:

$$\text{Observable universe} = \text{Image}(P)$$

$$\text{Infinities} = \text{Null}(P)$$

The universe contains no infinities. Our descriptions sometimes do, when we push a representation beyond its admissible domain. The infinities are not being **subtracted** (as in renormalization folklore), or **avoided** (as in singularity theorems), or **regularized** (as in the standard formalism). They were never there. They are null-space elements of particular representations — the mathematical equivalent of the Mercator projection's infinite poles.

The Hadamard projection is not a **procedure** applied to a sick theory to extract finite answers. It is the **definition** of what it means to extract a physical observable from a formal representation.

We state this as a formal postulate:

Postulate (Observability as Projection). Physical observables correspond to elements of a formal series that are stable under the admissibility projection. That is, a quantity O is observable if and only if $P(O) = O$.

This postulate implies: every observable is in the image of P; every divergence is in the null space of P; and the decomposition $f = P(f) + (f - P(f))$ separates the physical content from the representational artifact.

This completes the POT program:

- **Volumes I–III**: Physical laws are properties of admissible kernels.
- **Volume IV**: Non-admissibility of the kernel produces structural confinement.
- **Volume V**: Admissibility restoration produces the structural Higgs.
- **Volume VI**: The four-sentence axiom.
- **Volume VII**: Renormalization is a projection kernel; the ITCM composite kernel.
- **Volume VIII**: Conditional reduction of the Yang–Mills mass gap problem via ITCM.

- **Volume IX**: Yang–Mills vacuum stability as a classical spectral property.
- **Volume X**: All infinities are projection singularities.

A natural objection: is the Hadamard projection one filter among many, or is it uniquely forced? The answer (Theorem 1b) is that given an admissibility partition, the Hadamard projection is the **unique** linear idempotent respecting it. And the partition itself is not declared but constrained: the admissible indices are exactly those whose series contributions remain bounded, measurable, and stable under the observable limit (Section 2). The chain of determination is: analytic structure of the representation → admissibility partition → unique projection → observable. No step is free.

There is nothing left to renormalize, no singularity to resolve. The representation has a boundary; the physics does not.

All 29 structural results in this paper are machine-verified by the Z3 SMT solver, and the worked examples (A_0 and B_0) are verified by direct computation, in the Kleis formal verification language.

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