

# Confinement as Fiber Non-Invariance: The Admissibility Boundary in Projected Ontology

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## Abstract

We show that color confinement admits a structural necessary condition rooted in the non-admissibility of the Yang–Mills projection kernel, independent of quantum-mechanical postulates. In the Projected Ontology (POT) framework, observable physics arises from applying a projection kernel  $K$  to a pre-observable configuration space. Volumes I–III established that flat galactic rotation curves, Bell inequality violation, and classical electrodynamics all follow from admissible kernels — kernels satisfying linearity and zero preservation. The electromagnetic kernel  $K_{\text{em}} = d$  is admissible because  $U(1)$  is abelian; its image (the field strength  $F = dA$ ) is gauge-invariant, making electric charge a projective observable. The Yang–Mills kernel  $K_{\text{YM}}(A) = dA + A \wedge A$  is not admissible: the quadratic self-interaction  $A \wedge A$  breaks linearity. We identify the admissibility defect  $\Delta(A, B) = K(A + B) - K(A) - K(B)$  with the Lie bracket  $[A, B]$  of the gauge algebra, establishing that admissibility fails if and only if the gauge group is non-abelian. Non-admissibility implies that the kernel’s image is non-invariant on gauge orbits (fibers of the projection). By the Main Theorem of projection fiber theory — independence if and only if non-invariance — color charge becomes a fiber-non-invariant predicate: it cannot be determined from gauge-invariant observables alone. We define this structural unobservability as **confinement within the POT framework**, and show that it constitutes a necessary algebraic condition for the dynamical confinement observed in QCD. The same structural mechanism yields the Gribov obstruction (no global gauge fixing for non-affine orbits), the observable hierarchy (minimum second-order gauge-invariant observables for non-abelian theories), and the nonlinearity of the moduli space of flat connections. The derivation does not address the linear confining potential, the mass gap, or asymptotic freedom — these require dynamical content beyond the algebraic structure formalized here. All results are verified for internal logical consistency by the Z3 SMT solver via 19 axiomatic examples. The structural pattern parallels the Cantor independence result: the Continuum Hypothesis is independent of cardinal arithmetic for the same reason — both are non-invariant predicates on fibers of a non-injective projection.

**Keywords:** confinement, Yang-Mills, admissible kernel, projected ontology, Lie bracket, fiber non-invariance, gauge theory, Gribov problem, formal verification, Z3 theorem prover

## 1 Introduction

Color confinement — the impossibility of isolating free quarks or gluons — is one of the deepest phenomena in fundamental physics. In the standard framework, confinement is understood

dynamically: the running coupling constant of quantum chromodynamics (QCD) grows at low energies, producing a linear confining potential between color charges. This understanding rests on non-perturbative methods (lattice QCD, Dyson–Schwinger equations, the Gribov–Zwanziger framework) and has not been derived from first principles. The Yang–Mills mass gap problem — proving that the quantum theory has a positive lower bound on its spectrum — remains one of the seven Millennium Prize Problems (Jaffe and Witten, 2000). For a comprehensive review of the confinement problem and the major approaches, see Greensite (2011).

This paper identifies an algebraic **necessary condition** for confinement that is independent of quantum-mechanical postulates: the non-admissibility of the projection kernel. We work within the Projected Ontology (POT) framework, in which observable physics arises from applying a **projection kernel** to a pre-observable configuration space. The only assumption about gauge theory is the algebraic structure of the kernel: the Yang–Mills kernel  $K_{\text{YM}}(A) = dA + A \wedge A$  includes a quadratic self-interaction  $A \wedge A$  that the electromagnetic kernel  $K_{\text{em}}(A) = dA$  does not.

From this single algebraic difference, we derive five structural results:

1. **The Abelian Classification (Theorem 1).** The kernel is admissible (linear) if and only if the gauge group is abelian. This connects the POT framework to the Cartan–Killing classification of Lie algebras.
2. **The Confinement Theorem (Theorem 3).** Non-admissible kernels have images that are non-invariant on gauge orbits (fibers). By the Main Theorem of projection fiber theory (proven in the companion paper on independence), non-invariant predicates are **independent** of the projected observables. Color charge — which couples to the non-invariant image — cannot be determined from gauge-invariant observation. We call this **structural confinement**: the algebraic precondition that makes dynamical confinement possible.
3. **The Gribov Obstruction (Theorem 4).** Admissible kernels produce affine gauge orbits admitting global sections (gauge fixing). Non-admissible kernels produce non-affine orbits where no global section exists. This is the Gribov ambiguity, derived from the algebra of the kernel.
4. **The Observable Hierarchy (Theorem 5).** For admissible kernels, the field strength  $F$  itself is a gauge-invariant observable (order 1). For non-admissible kernels, only quadratic combinations  $\text{Tr}(F \wedge F)$  survive as gauge invariants (order  $\geq 2$ ).
5. **Nonlinear Nullspace (Theorem 6).** The nullspace of an admissible kernel is a vector subspace. The nullspace<sup>‘</sup> of a non-admissible kernel — the moduli space of flat connections — is a nonlinear variety.

All five results flow from a single source: the admissibility defect  $\Delta(A, B) = K(A + B) - K(A) - K(B)$ , which we identify with the Lie bracket  $[A, B]$  of the gauge algebra. The defect is zero if and only if the gauge group is abelian. The defect is the obstruction.

We emphasize what this paper does **not** do. It does not derive the linear confining potential (area law for Wilson loops), the mass gap, asymptotic freedom, or the hadron spectrum. These are dynamical phenomena requiring structure beyond the algebraic framework developed here. What we establish is the **structural layer** beneath these phenomena: the algebraic obstruction that makes confinement possible in non-abelian theories and impossible in abelian ones.

This paper is the fourth in a series on Projected Ontology Theory. The first derived flat galactic rotation curves from the admissible kernel framework without dark matter. The second derived Bell inequality violation and GHZ contextuality from the same framework, without quantum postulates. The third derived classical electrodynamics from the kernel  $K_{\text{em}} = d$  and identified it as the unique admissible gauge sector — noting that Yang–Mills breaks admissibility, a direction for future work.’ The present paper takes that step. It also connects to the companion projection fibers papers, which proved that the Continuum Hypothesis is independent of cardinal arithmetic for the same structural reason: non-invariance on fibers of a non-injective projection.

Every derivation is formally verified for internal logical consistency. The axioms are explicit, stated in the Kleis verification language, and checked by the Z3 SMT solver. The complete theory file (`theories/pot_yang_mills_confinement.kleis`) contains 11 structures and 19 verified examples.

## 2 The Projected Ontology Framework

We recall the three components of the POT framework established in Volumes I–III, then introduce the fiber non-invariance machinery from the companion papers.

### 2.1 Admissible Kernels

The POT framework posits an ontological configuration space (the space of **flows**) and an observable field space. A **projection kernel**  $K$  maps flows to fields. A kernel is **admissible** if it satisfies:

- **Additivity:**  $K(a + b) = K(a) + K(b)$  for all flows  $a, b$       (`kernel_lin_add`)
- **Scalar linearity:**  $K(c \cdot a) = c \cdot K(a)$  for all scalars  $c$       (`kernel_lin_smul`)
- **Zero preservation:**  $K(0) = 0$       (`kernel_maps_zero`)

These axioms are formalized in `pot_admissible_kernels_v2.kleis`. The admissible kernel framework also defines **projective equivalence** ( $a \sim b$  iff  $K(a) = K(b)$ ), **nullspace** ( $a \in \ker(K)$  iff  $K(a) = 0$ ), and **composition closure** (the composition of admissible kernels is admissible).

Volume I identified the gravitational kernel as a logarithmic Green’s function, deriving flat rotation curves. Volume II factorized the kernel as  $K = K_{\text{univ}} \circ K_{\text{dyn}} \circ K_{\text{rep}}$  and derived Bell violation from the measurement sector  $K_{\text{rep}}$ . Volume III identified the electromagnetic kernel  $K_{\text{em}} = d$  (the exterior derivative) as admissible, and noted that the Yang–Mills kernel breaks admissibility.

### 2.2 Projection Fibers and Non-Invariance

The companion papers on projection fibers formalized a universal pattern. Given a projection  $\Pi : \mathcal{O}_{\text{ont}} \rightarrow \mathcal{O}_{\text{obs}}$ , the **fiber** over an observation  $o$  is the preimage  $\text{Fib}(o) = \Pi^{-1}(o)$  — all ontological states that produce the same observable.

A predicate  $P$  on the ontological layer is **fiber-invariant** if it takes the same value on all fiber-mates:  $\Pi(m_1) = \Pi(m_2) \Rightarrow (P(m_1) \leftrightarrow P(m_2))$ .

The **Main Theorem** (machine-verified):

### Independence $\leftrightarrow$ Non-Invariance

A predicate is independent of the projected theory if and only if it is non-invariant on fibers. The Continuum Hypothesis is independent of cardinal arithmetic because it is non-invariant on fibers of the Skolem constraint map: models  $M_{\text{CH}}$  and  $M_{\neg \text{CH}}$  project to the same cardinal arithmetic but disagree on CH.

This theorem applies to **any** non-injective projection. In this paper, we instantiate it for gauge theory: the projection is the kernel  $K$ ; the fibers are gauge orbits; and the non-invariant predicate is color charge.

### 3 The Admissibility Defect

For any kernel  $K$ , define the **admissibility defect**:

$$\Delta(K, A, B) = K(A + B) - K(A) - K(B)$$

This measures the failure of additivity. For an admissible kernel,  $\Delta = 0$  identically (this is the content of `kernel_lin_add`). For a non-admissible kernel,  $\Delta \neq 0$  for some inputs  $A, B$ .

For the Yang–Mills kernel  $K_{\text{YM}}(A) = dA + A \wedge A$ , the defect computes to:

$$\begin{aligned} \Delta(A, B) &= [d(A + B) + (A + B) \wedge (A + B)] - [dA + A \wedge A] - [dB + B \wedge B] \\ &= dA + dB + A \wedge A + A \wedge B + B \wedge A + B \wedge B - dA - A \wedge A - dB - B \wedge B \\ &= A \wedge B + B \wedge A \end{aligned}$$

For Lie-algebra-valued 1-forms,  $A \wedge B + B \wedge A = [A, B]$ , the **Lie bracket** of the gauge algebra. Therefore:

$$\text{The admissibility defect is the Lie bracket: } \Delta(A, B) = [A, B]$$

This is an algebraic identity, not a physical hypothesis. It connects the POT framework directly to the structure theory of Lie algebras:

- **Abelian gauge group** (e.g.  $U(1)$ ):  $[A, B] = 0$  for all  $A, B \Rightarrow \Delta = 0 \Rightarrow$  kernel is admissible.
- **Non-abelian gauge group** (e.g.  $SU(2), SU(3)$ ):  $[A, B] \neq 0$  for generic  $A, B \Rightarrow \Delta \neq 0 \Rightarrow$  kernel is not admissible.

The defect inherits the algebraic properties of the Lie bracket: **antisymmetry** ( $\Delta(A, B) = -\Delta(B, A)$ ) and **self-annihilation** ( $\Delta(A, A) = 0$ ). It vanishes identically if and only if the gauge algebra is abelian.

### 4 The Abelian Classification

We now state and verify the classification theorem.

**Theorem 1 (Abelian Classification).** A projection kernel of Yang–Mills type is admissible if and only if the gauge group is abelian.

**Proof.** ( $\Rightarrow$ ) If the gauge group is abelian, the Lie bracket vanishes:  $[A, B] = 0$  for all  $A, B$ . Therefore  $\Delta(A, B) = 0$ , and the kernel satisfies `kernel_lin_add`. The other admissibility axioms (`kernel_lin_smul`, `kernel_maps_zero`) are satisfied by the linear part  $d$ . Hence  $K$  is admissible.

( $\Leftarrow$ ) If the gauge group is non-abelian, there exist  $A, B$  with  $[A, B] \neq 0$ . Therefore  $\Delta(A, B) \neq 0$ , and `kernel_lin_add` fails. Hence  $K$  is not admissible.  $\square$

This classification connects gauge theory to the POT framework via a single invariant: the Lie bracket. Abelian gauge groups ( $U(1)$ , products of  $U(1)$ ) produce admissible kernels. Non-abelian gauge groups ( $SU(N)$ ,  $SO(N)$ , exceptional groups) produce non-admissible kernels. The boundary is sharp and algebraic.

The classification is verified by Z3 in three examples: `abelian`  $\square$  `admissible`, `non-abelian`  $\square$  `non-admissible`, and the biconditional classification: `admissible iff abelian`.

## 5 Gauge Orbits as Projection Fibers

A gauge transformation  $g$  acts on a connection (flow)  $A$  by  $A \rightarrow A^g$ . The **gauge orbit** of  $A$  is the set of all gauge-equivalent connections:

$$\mathcal{O}(A) = \{A^g : g \in \mathcal{G}\}$$

where  $\mathcal{G}$  is the gauge group. In the POT fiber language:

- The ontological states are connections  $A$  (flows).
- The projection is the kernel  $K$ .
- The fiber over an observation is the gauge orbit: all connections producing the same observable.

For **abelian** gauge theory ( $U(1)$ ), the gauge transformation is  $A \rightarrow A + d\chi$ , and the orbit is:

$$\mathcal{O}(A) = A + \{d\chi : \chi \in \Omega^0(M)\} = A + \ker(K_{\text{em}})$$

This is an **affine subspace** — a coset of the kernel's nullspace. The geometry is flat (Euclidean).

For **non-abelian** gauge theory ( $SU(N)$ ), the gauge transformation is  $A \rightarrow g^{-1}Ag + g^{-1}dg$ , and the orbit is:

$$\mathcal{O}(A) = \{g^{-1}Ag + g^{-1}dg : g \in \mathcal{G}\}$$

This is a **nonlinear manifold** — not a vector subspace, not an affine space, not flat. The geometry is curved, and the topology can be non-trivial.

The structural difference is determined by admissibility: admissible kernels produce affine fibers; non-admissible kernels produce curved fibers. This is formalized in `GaugeOrbitFiber` and `ConcreteGaugeOrbit` in the theory file, with Z3-verified examples confirming that gauge-equivalent connections lie in the same fiber and are ontologically distinct.

## 6 Projective Stability: Image Invariance on Fibers

The central structural distinction of this paper is **projective stability**: whether the kernel's image is constant along the fibers.

**Definition.** A kernel  $K$  is **projectively stable** if its image is invariant on gauge orbits:

$$K(A^g) = K(A) \quad \forall g \in \mathcal{G}, \forall A$$

**Theorem 2 (Projective Stability).** A kernel is projectively stable if and only if it is admissible.

**Proof.** ( $\Rightarrow$ ) Let  $K$  be admissible. For  $U(1)$ , the gauge-transformed connection is  $A^g = A + d\chi$ . By additivity:  $K(A + d\chi) = K(A) + K(d\chi) = K(A) + d^2\chi = K(A) + 0 = K(A)$ . The image is invariant. (The key step uses admissibility to factor the sum, then  $d^2 = 0$  to annihilate the gauge term.)

( $\Leftarrow$ ) Let  $K$  be non-admissible. The gauge transformation  $A \rightarrow g^{-1}Ag + g^{-1}dg$  is nonlinear in both  $g$  and  $A$ . The non-admissibility of  $K$  means we cannot factor  $K(A^g)$  into  $K(A)$  plus corrections. In general,  $K(A^g) = g^{-1}K(A)g \neq K(A)$  — the image **transforms** in the adjoint representation.  $\square$

The physical consequence:

- **Admissible kernel:**  $F = dA$  is gauge-**invariant**. Two observers in different gauges see the **same**  $F$ . The field strength is a projective observable.
- **Non-admissible kernel:**  $F = dA + A \wedge A$  is gauge-**covariant**. Two observers in different gauges see **different**  $F^a$ . The field strength is **not** a projective observable.

This is formalized in `ProjectiveStability` with Z3-verified examples for both cases.

## 7 The Confinement Theorem

We now combine the Abelian Classification (Theorem 1), Projective Stability (Theorem 2), and the Main Theorem of projection fiber theory to derive structural confinement.

**Definition.** A **charge** is a property of the ontological state (connection  $A$ ) that couples to the kernel's image. Color charge is defined by the components  $F^a$  of the field strength in the gauge algebra basis.

**Definition (Structural Confinement).** A charge is **structurally confined** with respect to a projection kernel  $K$  if it is a fiber-non-invariant predicate: it varies across gauge orbits and therefore cannot be determined from gauge-invariant measurements alone. Structural confinement is an algebraic necessary condition for the dynamical confinement observed in QCD. It establishes that the **possibility** of free color charges is algebraically excluded by the kernel's non-admissibility, independent of any dynamical mechanism (running coupling, flux tubes, area law) that governs **how** confinement is enforced.

**Theorem 3 (Structural Confinement).** If the projection kernel is non-admissible, then color charge is structurally confined.

**Proof.** The argument proceeds in three steps:

**Step 1.** The kernel is non-admissible (Theorem 1: because the gauge group is non-abelian, and the Lie bracket  $[A, B] \neq 0$ ).

**Step 2.** The kernel's image  $F^a$  is non-invariant on fibers (Theorem 2: non-admissible  $\Rightarrow F^a$  transforms as  $F^a \rightarrow g^{-1}F^a g$ , varying across the gauge orbit).

**Step 3.** By the Main Theorem of projection fiber theory:

$$\text{Non-invariant on fibers} \leftrightarrow \text{Independent of observables}$$

Since  $F^a$  is non-invariant on fibers, color charge — which is defined by  $F^a$  — is **independent of the gauge-invariant observables**. It cannot be measured. It is structurally confined.  $\square$

**Remark on the relationship to dynamical confinement.** The standard notion of confinement in QCD involves specific dynamical content: the area law for Wilson loops, the linear potential  $V(r) \sim \sigma r$ , and asymptotic freedom. These are properties of the quantum dynamics, not of the algebraic structure alone. Structural confinement as defined here is strictly weaker: it establishes that no algebraic mechanism exists to extract color charge from gauge-invariant observables. This is necessary but not sufficient for dynamical confinement — one also needs the dynamics to prevent color-singlet screening from trivializing the constraint. The relationship is analogous to symmetry and symmetry breaking: the group structure (algebra) determines **which** breakings are possible; the Hamiltonian (dynamics) determines **which** actually occur.

The theorem establishes structural confinement as the gauge-theoretic instance of a universal pattern. The same structural argument proves that the Continuum Hypothesis is undecidable:

Component	Cantor (CH)	Yang–Mills (Color)
Ontology	ZFC models	Connections $A$
Projection	Cardinal shadow $\Pi$	Kernel $K(A)$
Observable	Cardinal arithmetic	Gauge-invariant combos
Fiber	Models with same shadow	Gauge orbit $\{A^g\}$
Distinguisher	CH holds?	Color charge
Non-invariance	CH varies on fiber	Color varies on fiber
Conclusion	CH undecidable	Color confined

Both are instances of: **a non-invariant predicate on fibers of a non-injective projection cannot be determined from the projection's image.**

The difference: for Cantor, the non-injectivity comes from the shadow dropping set-theoretic structure. For Yang–Mills, the non-injectivity comes from the kernel's self-interaction  $A \wedge A$  destroying linearity. In both cases, the obstruction is the **kernel** — the gap between what exists and what is observed.

The confinement theorem is verified by Z3 in examples CONFINEMENT THEOREM: non-admissible  $\square$  charge unobservable and free charges: admissible  $\square$  charge observable, and the combined CLASSIFICATION: abelian  $\square$  free, non-abelian  $\square$  confined.

## 8 Structural Consequences

The non-admissibility of the Yang–Mills kernel produces four additional structural results, all verified by Z3.

### 8.1 The Gribov Obstruction

**Theorem 4 (Gribov).** An admissible kernel admits global gauge fixing; a non-admissible kernel does not.

For an admissible kernel, the gauge orbit is an affine subspace  $A + \ker(K)$ . A global section exists: choose a complementary subspace  $V$  with  $\Omega^1 = \ker(K) \oplus V$ , and select the unique representative in  $V$  from each orbit. This is gauge fixing (e.g. Coulomb gauge  $\nabla \cdot A = 0$  for  $U(1)$ ).

For a non-admissible kernel, the gauge orbit is a nonlinear manifold. A global section may not exist: the gauge-fixing surface can intersect each orbit more than once, producing **Gribov copies**. This was discovered by Gribov in 1978 and is a major obstacle to the quantization of non-abelian gauge theories.

In the POT framework, the Gribov problem is a **theorem of non-admissibility**: non-affine fibers need not admit global sections. The obstruction is algebraic, not dynamical.

### 8.2 The Observable Hierarchy

**Theorem 5 (Observable Order).** For an admissible kernel, the field strength  $F$  is directly observable (order 1). For a non-admissible kernel, the minimum gauge-invariant observable has order 2.

When  $K$  is admissible,  $F = dA$  is gauge-invariant: it does not change under  $A \rightarrow A + d\chi$ . Any linear functional of  $F$  is an observable.

When  $K$  is non-admissible,  $F = dA + A \wedge A$  transforms as  $F \rightarrow g^{-1} F g$ . No linear functional of  $F^a$  is gauge-invariant. The simplest invariants are **quadratic**:

$$\text{Tr}(F \wedge F), \quad \text{Tr}(F \wedge \star F)$$

These require **two** copies of  $F$  and a trace over the gauge algebra. No first-order (linear in  $F$ ) gauge-invariant exists.

The physical consequence: non-abelian gauge theories have **lower observational resolution** than abelian ones. You cannot see ‘the individual field components  $F^a$  — only their collective, traced combinations. This is the algebraic manifestation of confinement at the level of observables.

### 8.3 Nonlinear Nullspace

**Theorem 6 (Nullspace Geometry).** The nullspace of an admissible kernel is a vector subspace; the nullspace of a non-admissible kernel is a nonlinear variety.

For an admissible kernel,  $a, b \in \ker(K) \Rightarrow K(a+b) = K(a) + K(b) = 0 + 0 = 0$ , so  $a+b \in \ker(K)$ . The nullspace is closed under addition — it is a vector subspace.

For a non-admissible kernel, the nullspace’s  $\{A : dA + A \wedge A = 0\}$  is the **moduli space of flat connections**. This is not closed under addition: if  $dA + A \wedge A = 0$  and  $dB + B \wedge B = 0$ , it does not follow that  $d(A+B) + (A+B) \wedge (A+B) = 0$ , because of the cross terms  $A \wedge B + B \wedge A = [A, B]$ .

The moduli space of flat connections is a central object in mathematical gauge theory. It carries a natural symplectic structure (the Atiyah–Bott construction) and its topology encodes invariants of the underlying manifold. In the POT framework, this rich geometry is a **consequence of non-admissibility** — the kernel’s self-interaction endows the nullspace with nonlinear structure that the admissible case lacks entirely.

**Remark (Nullspace topology and the mass gap).** The contrast between admissible and non-admissible nullspaces may be relevant to the mass gap problem. An admissible nullspace is a vector subspace — contractible and topologically trivial. Fluctuations around zero can be made arbitrarily small; no topological constraint forces a spectral gap. A non-admissible nullspace is a nonlinear variety whose topology can be non-trivial: for  $SU(2)$  on a Riemann surface of genus  $g$ , the moduli space of flat connections has dimension  $3(g-1)$  and non-trivial fundamental group and higher homology. Non-trivial topology constrains the spectrum of excitations: fluctuations cannot continuously shrink through a topological obstruction, suggesting that a discrete (gapped) spectrum may be a **topological consequence** of nullspace nonlinearity. This is consistent with the role of instantons and theta vacua in standard Yang–Mills theory, where the topology of the configuration space ( $\pi_3(SU(N)) = \mathbb{Z}$ ) creates distinct topological sectors that affect the spectrum. Whether the nullspace topology **forces** a mass gap — i.e., whether spectral geometry on the moduli space variety produces a positive spectral lower bound — is an open question that we do not resolve here, but it suggests that the mass gap may ultimately be a structural feature of non-admissibility rather than a purely dynamical phenomenon.

## 8.4 Connection to Standard Observables

The structural results of Theorems 1–6 are algebraic. We now show that they map directly onto four central constructs of standard gauge theory: Wilson loops, the Gauss law constraint, the representation-theoretic structure of confinement, and the observed phenomenology of deep inelastic scattering.

**Wilson loops as predicted observables.** Theorem 5 establishes that non-admissible kernels force the minimum gauge-invariant observable to be of order  $\geq 2$ : no linear functional of  $F^a$  is gauge-invariant. What **is** the simplest gauge-invariant observable in non-abelian gauge theory? It is the Wilson loop:

$$W(C) = \text{Tr } \mathcal{P} \exp \left( \oint_C A \right)$$

where  $\mathcal{P}$  denotes path-ordering and the trace is over the gauge algebra. The Wilson loop is gauge-invariant precisely because the trace kills the adjoint transformation:  $\text{Tr}(g^{-1}Mg) = \text{Tr}(M)$ . It is nonlinear in  $A$  — indeed, it is an infinite-order polynomial (the exponential series). In the POT framework, Wilson loops are the **canonical realization** of the observable hierarchy theorem: when the kernel is non-admissible, the theory is forced to construct observables from closed, traced, nonlinear composites of the connection. The area law for Wilson loops —  $W(C) \sim \exp(-\sigma \cdot \text{Area}(C))$  in confining theories — is a dynamical property that goes beyond our structural result, but the **existence** of Wilson loops as the natural observable is a structural prediction of Theorem 5.

For admissible (abelian) theories, the comparison is sharp: the field strength  $F = dA$  is itself gauge-invariant. No path-ordering or trace is needed. The electric and magnetic fields are directly observable. The Wilson loop simplifies to  $\exp(i \oint A) = \exp(i\Phi)$ , where  $\Phi$  is the magnetic flux — a quantity linear in  $A$ .

**The Gauss law as fiber-invariance.** In Hamiltonian gauge theory, the Gauss law constraint requires that physical states  $|\Psi\rangle$  satisfy:

$$D_i E^{ai} |\Psi\rangle = 0$$

where  $D_i$  is the covariant derivative and  $E^{ai}$  is the chromoelectric field. This constraint is equivalent to requiring that physical states are **gauge-invariant**: they must be invariant under all gauge transformations generated by the constraint.

In the POT framework, this is precisely the **fiber-invariance condition**. Physical observables are fiber-invariant predicates — they take the same value on all gauge-equivalent configurations (all points in the same fiber). The Gauss law is the standard formulation of what we call projective stability” (Theorem 2): only fiber-invariant quantities survive as observables.

For abelian theories ( $U(1)$ ), the Gauss law is  $\nabla \cdot \mathbf{E} = \rho$  — a linear constraint. It restricts the physical Hilbert space but does not confine electric charge: single electrons are gauge-invariant states.

For non-abelian theories ( $SU(N)$ ), the Gauss law  $D_i E^{ai} = 0$  is a **nonlinear** constraint (because  $D_i = \partial_i + g[A_i, \cdot]$  contains the connection). It restricts the physical Hilbert space to **color singlets** — states in the trivial representation of the gauge group. Single quarks (color triplets) or single gluons (color octets) are **not** gauge-invariant: they transform nontrivially under gauge transformations. This is structural confinement in the language of the Gauss law.

**Color representations and fiber non-invariance.** The representation-theoretic structure of confinement follows directly from Theorem 3. Under a gauge transformation  $g \in SU(N)$ :

- A quark field  $\psi$  transforms as  $\psi \rightarrow g\psi$  (fundamental representation)
- A gluon field  $A$  transforms as  $A \rightarrow g^{-1}Ag + g^{-1}dg$  (adjoint representation)
- The meson operator  $\bar{\psi}\psi$  transforms as  $\bar{\psi}\psi \rightarrow \bar{\psi}g^\dagger g\psi = \bar{\psi}\psi$  (singlet — invariant)
- The baryon operator  $\varepsilon^{abc}\psi_a\psi_b\psi_c$  transforms as a singlet for  $SU(3)$  (invariant)

In the POT language: quark color and gluon color are **fiber-non-invariant predicates** — they vary across the gauge orbit. Only color-singlet combinations are fiber-invariant. This is exactly the content of Theorem 3 (structural confinement), stated in the language of representation theory. The structural result predicts: for any non-abelian gauge group, **only singlet representations**

are **gauge-invariant**, and therefore only singlets are observables. Quarks (non-singlets) cannot appear as asymptotic states.

For  $U(1)$ : every representation is one-dimensional and gauge-invariant (the phase cancels in  $|\psi|^2$ ). Single charged particles are observable. This is the admissible case.

**Deep inelastic scattering and the two-kernel structure.** High-energy electron-proton collisions provide the most direct experimental window into proton structure. In these experiments, a virtual photon probes the proton’s constituents, revealing point-like objects with fractional electric charges ( $2/3$  for up quarks,  $-1/3$  for down quarks), measurable momentum fractions, and definite spin. Decades of precision measurements — from the original SLAC experiments through HERA, JLab, and the forthcoming Electron-Ion Collider — have mapped this internal structure in extraordinary detail. Any structural theory of confinement must be compatible with these results.

Does the existence of observable proton substructure contradict structural confinement? It does not. The resolution lies in the **two-kernel structure** that our framework makes explicit. The electron probes quarks through the **electromagnetic** interaction, whose kernel  $K_{\text{em}} = dA$  is admissible (abelian,  $U(1)$ ). Electric charge is a fiber-invariant predicate under this kernel, and is therefore observable — this is why experiments can measure the fractional charges  $2/3$  and  $-1/3$ . But the same constituents also carry a charge associated with a non-abelian gauge kernel  $K_{\text{YM}} = dA + A \wedge A$ , which is non-admissible. Whatever property this non-admissible kernel governs — call it ‘color’ — is a fiber-non-invariant predicate and is therefore structurally unobservable. The standard framework identifies this kernel with the strong interaction; the POT framework derives the unobservability from the kernel’s algebraic structure alone.

The experimental situation is precisely what the observable hierarchy predicts: the quantities measured in deep inelastic scattering — structure functions  $F_1(x, Q^2)$ ,  $F_2(x, Q^2)$  — are gauge-invariant composites (products of quark fields contracted into color singlets). They are order-2 observables in the sense of Theorem 5. No experiment has ever measured which **color** a specific quark carries, because color is not a gauge-invariant quantity. What is observed is the quarks’ electromagnetic properties (admissible kernel  $\Rightarrow$  observable) while their color properties (non-admissible kernel  $\Rightarrow$  unobservable) remain structurally inaccessible. Deep inelastic scattering is, in the POT framework, a natural demonstration of the admissibility boundary: two kernels acting on the same constituents, one admissible and one not, producing one observable charge and one unobservable charge.

**Remark (Energy independence of structural confinement).** The structural claim is energy-independent. The non-admissibility of the Yang–Mills kernel is a property of the gauge algebra —  $SU(N)$  is non-abelian, and the Lie bracket  $[A, B]$  does not vanish — not of the coupling constant or the energy scale. This predicts that color remains unobservable at arbitrarily high energies. In contrast, the dynamical aspects of confinement (the string tension, the flux tube profile, the confining potential) depend on the coupling and evolve with the energy scale. At high energies, asymptotic freedom weakens the coupling: quarks interact weakly at short distances and behave nearly as free particles. One might expect that sufficient energy could liberate ‘color’. But no isolated color charge has been observed at any energy scale ever tested. The structural explanation is that there is no energy threshold to overcome: the unobservability of color is algebraic, not dynamical. The kernel does not become admissible at high energy — the Lie bracket of  $SU(3)$

does not switch off. This distinguishes structural confinement from a confining **force**: a force can in principle be overcome by sufficient energy; an algebraic obstruction cannot.

**The quark-gluon plasma and the persistence of structural confinement.** The deconfinement phase transition — in which QCD matter transitions from hadronic to quark-gluon degrees of freedom at a critical temperature  $T_c \approx 155$  MeV — is real and well-established experimentally (RHIC at Brookhaven, LHC at CERN). In this transition, the linear confining potential disappears and quarks and gluons become the effective degrees of freedom. This is **dynamical** deconfinement: the force-based confinement mechanism ceases to operate. However, the gauge algebra does not change at the phase transition.  $SU(3)$  remains non-abelian above  $T_c$ . The Lie bracket  $[A, B]$  does not vanish. The kernel  $K = dA + A \wedge A$  remains non-admissible. In the POT framework, this means that **structural confinement persists in the quark-gluon plasma**: color remains a fiber-non-invariant predicate, and no QGP observable directly measures which color a specific quark carries. What is measured — collective flow, jet quenching, thermal radiation, energy density, shear viscosity — consists entirely of gauge-invariant quantities. The quark-gluon plasma is characterized by gauge-invariant thermodynamic observables, not by color assignments. The two-layer distinction is therefore: dynamical confinement (linear potential, flux tubes) breaks at the phase transition; structural confinement (unobservability of color) does not. The deconfinement transition changes **which degrees of freedom are dynamically relevant**; it does not change **which predicates are fiber-invariant**.

These four connections show that structural confinement (Theorem 3), the observable hierarchy (Theorem 5), projective stability (Theorem 2), and the two-kernel structure are not abstract constructions: they are the POT formulation of Wilson loops, the Gauss law, the singlet constraint, and the observed phenomenology of proton structure — the central pillars of the standard understanding of confinement.

## 9 Discussion

We address what this paper does and does not achieve, and its relation to the standard framework.

### 9.1 What We Derive

From a single algebraic property — the non-admissibility of the Yang–Mills kernel, caused by the Lie bracket  $[A, B]$  — we derive:

1. The **Abelian Classification**: admissible  $\leftrightarrow$  abelian (Theorem 1).
2. **Confinement**: non-admissible  $\Rightarrow$  charge unobservable (Theorem 3).
3. The **Gribov Obstruction**: non-admissible  $\Rightarrow$  no global gauge fixing (Theorem 4).
4. The **Observable Hierarchy**: non-admissible  $\Rightarrow$  minimum order 2 (Theorem 5).
5. **Nonlinear Nullspace**: non-admissible  $\Rightarrow$  moduli space, not subspace (Theorem 6).

Each result follows from the admissibility boundary. No quantum mechanics is assumed. No dynamics (Lagrangians, path integrals, running couplings) are invoked. The results are structural: they follow from the **algebra** of the kernel, not the **physics** of any specific theory.

## 9.2 What We Do Not Derive

This paper does **not** derive:

- The **linear confining potential** (area law for Wilson loops). Our confinement theorem establishes that color charge is unobservable, but does not specify the functional form of the confining interaction. The linear potential requires additional structure — likely the scale dependence of the effective kernel — that we do not formalize here.
- The **mass gap**. The Yang–Mills mass gap problem asks whether the quantum Hamiltonian has a positive spectral lower bound. Our framework is structural, not spectral. We conjecture that the non-admissibility of the kernel is consistent with a mass gap (the minimum observable order being 2 suggests a minimum energy scale), but we do not prove this.
- **Asymptotic freedom**. The running of the strong coupling constant  $\alpha_s$  with energy is a phenomenon that, in the standard framework, arises from quantum loop corrections. In the POT framework, scale-dependent behavior would require a scale-dependent kernel composition  $K = K_{\text{univ}} \circ K_{\text{dyn}} \circ K_{\text{rep}}$  where the measurement kernel  $K_{\text{rep}}$  resolves different scales. This is a direction for future work.
- The **hadron spectrum**. Deriving the masses and quantum numbers of specific bound states (protons, pions, glueballs) requires dynamics beyond the structural framework of this paper.

These limitations are honest. The paper contributes a **structural necessary condition** for confinement, not a complete dynamical theory.

## 9.3 Relation to Standard Approaches

The standard framework offers several confinement criteria, each operating at a different level:

- The **Kugo–Ojima criterion** is algebraic, formulated within BRST cohomology. Physical states are in  $\frac{\ker(s)}{\text{im}(s)}$  where  $s$  is the BRST charge. Confinement requires the ghost propagator to be infrared-enhanced. This is the closest existing precedent to our approach, but it operates within the **quantized** theory.
- The **Gribov–Zwanziger framework** restricts the path integral to the first Gribov region, modifying infrared Green functions. Our Gribov result (Theorem 4) is consistent with this framework: we derive the **existence** of Gribov copies from non-admissibility, while Gribov–Zwanziger addresses their **consequences** for the quantized theory.
- **Lattice QCD** computes confinement numerically via Monte Carlo simulation. It confirms the area law for Wilson loops and produces the hadron spectrum. Our result is complementary: it identifies the **algebraic origin** of the phenomenon that lattice QCD observes computationally.
- The **‘t Hooft classification** (‘t Hooft, 1978) uses center symmetry and magnetic monopole condensation. Our abelian/non-abelian classification (Theorem 1) is compatible with ‘t Hooft’s, but derived from kernel admissibility rather than center symmetry.
- **Functional methods** — Dyson–Schwinger equations and functional renormalization group (see Alkofer and von Smekal, 2001, for a review) — study confinement through the infrared

behavior of propagators and vertices. These approaches find positivity violations and infrared suppression in gluon propagators, which are dynamical signatures consistent with our structural result: a non-admissible kernel forces gauge-covariant (non-invariant) images, and the infrared behavior of propagators reflects this structural obstruction.

- The **Hamiltonian boundary approach** of Timoshenko (1995) derives confinement from the Gauss law constraint in Fock–Schwinger gauge. In the confinement phase, the chromo-electric flux through any boundary element is strictly zero, enforcing singletness with respect to the group of the residual gauge transformations and hence impossibility of observing coloured objects at spatial infinity.’ This is the dynamical realization of our structural confinement (Theorem 3): his Gauss law constraint is our fiber-invariance condition (Theorem 2), and his impossibility of observing coloured objects’ is our fiber-non-invariant predicate cannot be determined from observables.’ The correspondence is precise: his approach adds the dynamical content (partition function, mean-field approximation, confinement–deconfinement phase transition, Wilson loop area law) that we explicitly leave open. His result that the transition temperature scales correctly with the string tension in lattice Monte Carlo simulations provides independent evidence that the structural mechanism we identify — the Gauss law as fiber-invariance — is the correct algebraic substrate beneath the dynamical confinement.

The POT contribution is a **structural theorem** that sits upstream of these approaches. Non-admissibility is a necessary condition for all of them: the Kugo–Ojima, Gribov, lattice, ‘t Hooft, functional, and Hamiltonian boundary approaches all require a non-abelian gauge group, which is precisely the non-admissible case.

## 9.4 Falsifiability

The confinement theorem makes a falsifiable structural prediction:

**Prediction.** Any gauge theory with an admissible (abelian) projection kernel has observable charge carriers; any gauge theory with a non-admissible (non-abelian) kernel has unobservable charge carriers.

This predicts that no continuum abelian gauge theory (on a contractible manifold) will exhibit color-type confinement through the self-coupling mechanism, and that no pure non-abelian gauge theory will exhibit free colored asymptotic states. Both predictions are consistent with all known physics.

The prediction is structural, not dynamical: it does not address what happens when additional fields modify the effective kernel. Whether a non-admissible kernel can be **structurally restored** to admissibility by coupling to additional degrees of freedom — and what constraints such a restoration imposes on the restoring field — is a natural question for future work. If such a restoration mechanism exists within the POT framework, it would correspond to what the standard framework calls spontaneous symmetry breaking.

## 10 Conclusion

Structural confinement — the algebraic impossibility of extracting color charge from gauge-invariant observables — is the gauge-theoretic instance of a universal pattern: a non-invariant predicate on fibers of a non-injective projection cannot be determined from the projection’s image. The Continuum Hypothesis is undecidable for the same structural reason — both are fiber-non-invariant under a projection whose kernel destroys the information needed to resolve them.

The admissibility boundary — the transition from  $\Delta = 0$  (abelian, free) to  $\Delta = [A, B] \neq 0$  (non-abelian, structurally confined) — is the mechanism. From this single algebraic transition, five results follow: the abelian classification, structural confinement, the Gribov obstruction, the observable hierarchy, and the nonlinear nullspace. All are formally verified for internal logical consistency.

This paper does not close the Yang–Mills Millennium Prize problem. It does not derive the mass gap, the linear confining potential, the hadron spectrum, or asymptotic freedom. What it establishes is the **structural precondition** beneath all of these: the algebraic obstruction that makes confinement possible in non-abelian theories and impossible in abelian ones. The dynamics that build on this structure — and whether they produce a mass gap, a linear potential, or a specific spectrum — require additional content (scale-dependent kernels, dynamical composition) that we leave for future work.

The relationship between structural and dynamical confinement is analogous to the relationship between symmetry and symmetry breaking. The group structure determines which breakings are algebraically possible; the Hamiltonian determines which actually occur. Similarly, kernel non-admissibility determines that color charge **cannot** be a gauge-invariant observable; the dynamics determine **how** the theory enforces this constraint (flux tubes, screening, etc.). We have derived the algebraic ‘cannot’ — the dynamical ‘how’ — remains open.

The tools are available. The theory file `pot_yang_mills_confinement.kleis` is open source, machine-checkable, and extensible. The reader is invited to `kleis test` it.

## 10 References

- [pot-rotation] E. Atik, ‘Flat Galactic Rotation Curves as a Theorem of Projected Ontology,’ Kleis Research, 2025.
- [pot-entanglement] E. Atik, ‘Quantum Entanglement as a Projection Artifact,’ Kleis Research, 2025.
- [pot-electrodynamics] E. Atik, ‘Electrodynamics as a Theorem of Projected Ontology,’ Kleis Research, 2025.
- [projection-fibers] E. Atik, ‘Independence as Non-Invariance: Detecting Undecidability via Projection Fibers,’ Kleis Research, 2025.
- [fiber-dimension] E. Atik, ‘Observable Bounds on Ontological Dimension,’ Kleis Research, 2025.
- [gribov] V. N. Gribov, ‘Quantization of non-Abelian gauge theories,’ Nucl. Phys. B 139, 1–19 (1978).
- [kugo-ojima] T. Kugo and I. Ojima, ‘Local covariant operator formalism of non-Abelian gauge theories and quark confinement,’ Prog. Theor. Phys. Suppl. 66, 1–130 (1979).
- [yang-mills] C. N. Yang and R. L. Mills, ‘Conservation of isotopic spin and isotopic gauge invariance,’ Phys. Rev. 96, 191–195 (1954).
- [wilson] K. G. Wilson, ‘Confinement of quarks,’ Phys. Rev. D 10, 2445–2459 (1974).
- [thoof] G. ‘t Hooft, ‘On the phase transition towards permanent quark confinement,’ Nucl. Phys. B 138, 1–25 (1978).

- [atiyah-bott] M. F. Atiyah and R. Bott, ‘The Yang–Mills equations over Riemann surfaces,’ *Phil. Trans. R. Soc. A* 308, 523–615 (1983).
- [timoshenko] E. G. Timoshenko, ‘Boundary Effects and Confinement in the Theory of Nonabelian Gauge Fields,’ Ph.D. thesis, Moscow State University, D. V. Skobeltsyn Institute of Nuclear Physics (1995). arXiv:2405.13189.
- [jaffe-witten] A. Jaffe and E. Witten, ‘Quantum Yang–Mills Theory,’ Clay Mathematics Institute Millennium Prize Problem (2000).
- [alkofer] R. Alkofer and L. von Smekal, ‘The infrared behaviour of QCD Green’s functions,’ *Phys. Rep.* 353, 281–465 (2001). arXiv:hep-ph/0007355.
- [greensite] J. Greensite, *An Introduction to the Confinement Problem*, Lecture Notes in Physics 821, Springer (2011).
- [kleis] E. Atik, Kleis: A Formal Verification Language, <https://kleis.io>, 2024–2026.

# Appendix

## A Complete Axiom Inventory

The formal theory `pot_yang_mills_confinement.kleis` imports `pot_admissible_kernels_v2.kleis` and defines 11 structures:

1. **FieldNegation** — negation on the field algebra (3 axioms).
2. **AdmissibilityDefect** —  $\Delta(K, A, B) = K(A + B) - K(A) - K(B)$  (4 axioms).
3. **LieBracketAsDefect** —  $\Delta = [A, B]$  for non-abelian kernels (4 axioms).
4. **AbelianClassification** — admissible  $\leftrightarrow$  abelian, with witnesses (3 axioms, 2 elements).
5. **GaugeOrbitFiber** — gauge group action on connections (3 axioms, 2 elements).
6. **ConcreteGaugeOrbit** — witness pair of gauge-equivalent connections (4 axioms, 3 elements).
7. **ProjectiveStability** — image invariance on fibers (3 axioms, 2 elements).
8. **ConfinementTheorem** — charge observable iff admissible (3 axioms).
9. **GribovStructure** — global sections iff affine orbits iff admissible (3 axioms).
10. **ObservableHierarchy** — minimum observable order: 1 (admissible) or 2 (non-admissible) (2 axioms).
11. **NullspaceStructure** — nullspace closed under addition iff admissible (2 axioms).

Total: 34 axioms, 9 elements, 2 operations, 1 data type.

All 19 examples pass Z3 verification in under 25 seconds total.